COURSE PROPOSAL: FIRST COURSE IN LINEAR ALGEBRA

1. Course Title: First Course in Linear Algebra
2. Course Number: MTH2XX
3. Proposing Department: Mathematics and Statistics
4. Proposed by: Nandini Nilakantan, Preena Samuel
5. Units: 3-1-0-0 [11 credits]
6. Course Description:
   (A) Objectives of the course: A first course in linear algebra
   (B) Content (number of lectures mentioned in brackets):

   Review of Matrices: Operations on matrices, Elementary matrices, row reduction, RREF, expression of row and column operations via elementary matrices, systems of linear equations, Gauss-Jordan method, invertible matrices, inverse via RREF. Rank of a matrix, solutions of systems of equations. Permutations and definition of determinant via permutations, uniqueness and other properties of determinant; Cramer's Rule. (6)

   Vector spaces: Fields, Vector spaces over a field, subspaces, Linear independence and dependence, existence of basis, coordinates, dimension. (3)

   Linear Transformations: Kernel, image and rank of a transformation, Rank Nullity Theorem. (2)

   Isomorphism, matrix representation of linear transformation, change of basis, similar matrices, linear functional and dual space, dual basis. (3)

   Inner product spaces: Cauchy-Schwarz's inequality, Gram-Schmidt orthonormalization, orthonormal basis, Orthogonal matrices and rotations, matrix exponential. (3)

   Orthogonal projection, projection theorem, four fundamental subspaces and their relations (relation between null space and row space; relation between null space of the transpose and the column space). (2)

   Diagonalization: Eigenvalues and eigenvectors, diagonalizability. (2)

   Invariant subspaces, adjoint of an operator, normal, unitary and self adjoint operators, nilpotent operators, Schur's unitary triangularisation, diagonalization of normal matrices, spectral decompositions and spectral theorem, applications of spectral theorem.(3)

   Polynomial rings, Annihilating polynomials for linear transformations, Cayley-Hamilton theorem, minimal polynomials. (2)

Eigenspaces, generalised Eigenspaces, algebraic and geometric multiplicity, Jordan canonical form. (2)

Primary decomposition theorem, Rational canonical forms (with proofs). (3)

Introduction to the theory of Quadratic forms: Bilinear and quadratic forms, Sylvester's law of inertia. (4)

Tensor products, symmetric and exterior algebras, determinants via multilinear forms. (3)

(C) Pre-requisites: None

(D) Short summary: This is a compulsory course for MSc (MTH) students

7. Books/References:

(a) Kenneth Hoffman and Ray Kunze: Linear Algebra, PHI publication.

(b) Gilbert Strang: Linear Algebra and Its Applications, 4th edition.

(c) S. Lang: Algebra, GTM, Springer.

(d) M. Artin: Algebra, PHI publication.

Date: 28 March 2024 Signature of the Proposer:

Signature of DPGC Convener Mathematics and Statistics

This course is approved/not approved Chairman, SPGC