HEAT FLOW AT LARGE TIME ON CURVED SPACES

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If $f \in L^1(\mathbb{R}^n)$ with M denoting the integral of f then it is not hard to show that for all $p \in [1, \infty]$

$$\lim_{t \to \infty} \frac{\|f * h_t - M h_t\|_p}{\|h_t\|_p} = 0,$$

where $\{h_t \mid t > 0\}$ denotes the usual heat semigroup corresponding to the Laplace-Beltrami operator on $\mathbb{R}^n$. The main ingredient of the proof is the fact that h_t is a dilation of h_1. Analogs of the above result, for $p = 1$, has recently been proved by J. L. Vázquez (for real hyperbolic spaces) and J. P. Anker et al (for more general spaces). These are remarkable results because, unlike $\mathbb{R}^n$, on these spaces one does not have the advantage of using dilation. There are analogous results available for $p > 1$ also, but these results are weak analogs of the case $p = 1$. In this talk we will speak about the L^p version of this result for $p \in (1, 2]$, for simple spaces like $X = SL(2, R)/SO(2)$. The main point we will like to convey is that this L^p-analog is rooted in the so called Herz criteria for convolution operators.