FIM REGULARITY FOR GAUSSIAN SEMI-BLIND MIMO FIR CHANNEL ESTIMATION

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ABSTRACT

We consider the problem of Semi-Blind (SB) channel estimation for Multiple-Input Multiple-Output (MIMO) Finite Impulse Response (FIR) channels. We motivate a Fisher Information Matrix (FIM) based analysis of the semi-blind estimation problem and demonstrate that the semi-blind FIM can be written as the sum of a blind FIM J^B and training symbol FIM J^t. We show that the blind FIM J^B is rank deficient and establish the minimum number of training symbols necessary to achieve regularity (full-rank) of the FIM for identifiability. We also illustrate that an SB scheme can potentially be very efficient compared to an exclusively training based scheme since it estimates very few un-constrained parameters. It is then demonstrated that the rank deficiency of the FIM for an irreducible MIMO FIR channel arises because of a unitary (rotation) matrix indeterminacy. Based on this analysis a semi-blind algorithm is proposed for MIMO FIR channel estimation.

1. INTRODUCTION

MIMO (Multiple-Input Multiple-Output) communication systems have gained widespread popularity as technology solutions for current and future wireless systems. The performance of the designed MIMO decoders employed at the receiver and precoders employed at the transmitter depend critically on the accuracy of the available channel estimate. Semi-blind (SB) techniques have been suggested in [1] for channel estimation and trade off bandwidth efficiency for computational simplicity. Our work is an attempt to shed light on different aspects of SB estimation of a MIMO-FIR channel. First, assuming that the source symbols are Gaussian, we present the FIM for the SB estimation of a MIMO-FIR channel. We prove that FIM is rank deficient by at least the number of indeterminate parameters and demonstrate a series of results on the rank properties of the SB MIMO Gaussian FIM. Further, a novel contribution of our work has been to derive the change in the rank characteristics of the FIM for each additional pilot symbol transmitted and a result on the number of known (pilot) symbols that need to be transmitted for regularity or a full rank FIM. Also, the Gaussian covariance structure on the input symbols as compared to the deterministic symbol situation [2, 3] significantly changes the nature of the problem. The transmitted symbols are no more a deterministic unknown 'burden' but bear valuable statistical information. Thus the number of unknowns no longer grows with the number of transmitted blind symbols, making it possible to estimate the channel with only few transmitted symbols. We then motivate and utilize the irreducible-unitary decomposition for SB channel estimation. In what follows, $i \in \overline{m,n}$ represents $m \leq i \leq n$; $\quad i, m, n \in \mathbb{N}$, $rank\,(\cdot)$ the Rank of a matrix and $\mathcal{N}(\cdot)$ represents the Nullspace of a matrix. We formally set up the problem in the following section.

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2. PROBLEM SETUP

Consider an L tap MIMO Channel. Let the system input output relation be expressed as

$$\mathbf{y}(k) = \sum_{i=0}^{L-1} H(i)\mathbf{x}(k-i) + \mathbf{n}(k) \tag{1}$$

where $\mathbf{y}(k), \mathbf{x}(k)$ are the k^{th} received and transmitted symbol vectors respectively. Let t, r be the number of transmitters and receivers and therefore, $\mathbf{y}(k) \in \mathbb{C}^{r\times 1}$ and $\mathbf{x}(k) \in \mathbb{C}^{t\times 1}$. Each $H(i) \in \mathbb{C}^{r\times t}$, $i \in \overline{0, L-1}$ is the MIMO channel matrix corresponding to the i-th lag. Also, let $r > t$, i.e. the number of receivers is greater than the number of transmitters. Let $\{\mathbf{x}_p(1), \mathbf{x}_p(2), \ldots, \mathbf{x}_p(L_t)\}$ be a burst of L_t transmitted training symbols. Let $\mathbf{H} \in \mathbb{C}^{r\times Lt}$ be defined as

$$\mathbf{H} \triangleq [H(0), H(1), \ldots, H(L-1)]\,. \tag{2}$$

The input output relation can then be represented as $Y_p = \mathbf{H}X_p + N_p$, where the block Toeplitz pilot matrix $X_p \in \mathbb{C}^{Lt\times L_t}$ is constructed from the transmitted pilot symbols as

$$X_p \triangleq \begin{bmatrix} \mathbf{x}_p(1) & \mathbf{x}_p(2) & \dots & \mathbf{x}_p(L_t) \\ \mathbf{0} & \mathbf{x}_p(1) & \dots & \mathbf{x}_p(L_t-1) \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \dots & \mathbf{x}_p(L_t-L+1) \end{bmatrix}. \quad (3)$$

For the blind symbol transmissions, let us stack N received symbol vectors to define the channel matrix $\mathcal{H} \in \mathbb{C}^{Nr\times(L+N-1)t}$ as

$$\mathcal{H} \triangleq \begin{bmatrix} H(0) & H(1) & H(2) & \dots & 0 & \dots \\ 0 & H(0) & H(1) & \dots & H(L-1) & \dots \\ 0 & 0 & H(0) & \dots & H(L-2) & \dots \\ \vdots & \vdots & \vdots & \ddots & \vdots & \ddots \end{bmatrix} \quad (4)$$

Let $N \geq L$ for desirable rank properties of $\mathcal{H}$. The blind transmission block length P is defined as $P \triangleq N+L-1$. The input-output relation can then be written as $\mathcal{Y}(k) = \mathcal{H}\mathcal{X}(k) + \mathcal{N}(k)$, where the $\mathbb{C}^{Pt\times 1}$ stacked pilot symbol matrix $\mathcal{X}(k)$ is defined as $\mathcal{X}(k) \triangleq \left[\mathbf{x}\left((k+1)P\right)^T, \dots, \mathbf{x}\left(kP+1\right)^T\right]^T$. The block received vector $\mathcal{Y}(k)$ is similarly defined as $\mathcal{Y}(k) \triangleq \left[y\left((k+1)N\right)^T, \dots,\ y\left(kN+1\right)^T\right]^T$. The MIMO transfer function of the FIR channel can now be defined as $H(z) = \sum_{i=0}^{L-1} H(i)z^{-i}$. An important notion regarding such transfer functions is the concept of *irreducibility*. A MIMO transfer function $H(z)$ is said to be *irreducible* if $H(z)$ has full column rank for all $z \neq 0$ (but including $z=\infty$). It immediately follows that if $H(z)$ is irreducible, the leading coefficient matrix $H(0)$ has full column rank (substitute $z=\infty$ in $H(z)$). Irreducibility of the channel matrix is a key assumption for identifiability in several blind channel estimation algorithms [2]. In this work, we assume that the channel matrix $H(z)$ is irreducible. Let the transmitted blind data $\mathbf{x}(k)$ be spatio-temporally white, i.e. $\mathrm{E}\left\{\mathbf{x}(k)\mathbf{x}(l)^H\right\} = \sigma_s^2\delta(k,l)\mathbf{I}_{t\times t}$ and the normalized source power $\sigma_s^2 \triangleq 1$. Hence the covariance of the block input vector $\mathcal{X}(k)$ is given as $\mathcal{R}_{\mathcal{X}} \triangleq \mathrm{E}\left\{\mathcal{X}(i)\mathcal{X}(i)^H\right\} = \mathbf{I}_P$. Next, we present insights in to the nature of the above estimation problem.

3. SEMI-BLIND FISHER INFORMATION MATRIX (FIM)

In this section, we consider some properties of the semi-blind FIM and the resulting CRB matrix. We begin by describing some of the interesting properties of an FIM based analysis. Let $p\left(\bar{\omega}, g\left(\bar{\theta}\right)\right)$, be the pdf of the observation vector $\bar{\omega}$, parameterized by $\bar{\theta} \in \mathbb{C}^{m\times 1}$. Given the log-likelihood $\mathcal{L}\left(\bar{\omega},\bar{\theta}\right) \triangleq \ln p\left(\bar{\omega}, g\left(\bar{\theta}\right)\right)$, the FIM $J_{\bar{\theta}} \in \mathbb{C}^{m\times m}$ is given [2] as

$$J_{\bar{\theta}} \triangleq \frac{\partial^2 \mathcal{L}\left(\bar{\omega};\bar{\theta}\right)}{\bar{\theta}\bar{\theta}^H}. \quad (5)$$

Let $g\left(\bar{\theta}\right)$ be a many-to-one mapping, i.e. $g\left(\bar{\theta}\right) = f\left(\bar{\theta},\bar{\xi}\right) \forall \bar{\xi} \in \mathbb{C}^{r\times 1}$ or in other words, the function g remains unchanged as the parameter vector $\bar{\xi}$ varies over an r dimensional constrained manifold. $\bar{\xi}$ is then the **un-constrained parameterization** of the constraint manifold. The following lemma relates the number of such parameters to the rank of the Fisher Information Matrix (FIM).

Lemma 1. *If $g\left(\bar{\theta}\right) : \mathbb{C}^{n\times 1} \to \mathbb{C}^{m\times 1}$ and g is a many-to-one mapping, the FIM $J\left(\bar{\theta}\right) \in \mathbb{C}^{n\times n}$ is rank deficient and in fact, $rank\left(J\left(\bar{\theta}\right)\right) = n - r$.*

Proof. Given in [4]. □

Thus, the rank of the FIM is deficient by precisely the number of un-identifiable parameters as has been informally stated in [3]. In our analysis we examine the rank of the semi-blind FIM for several different cases and derive insights in to the nature of the estimation problem.

3.1. Semi-Blind Parameter Formulation

We now consider the FIM for the estimation of the channel matrix $\mathbf{H}$. Hence let us define the parameter vector to be estimated $\theta_{\mathcal{H}} \in \mathbb{C}^{2Lrt\times 1}$ by stacking the complex parameter vector and its conjugate as suggested in [5] as

$$\theta_{\mathcal{H}} \triangleq \begin{bmatrix} \theta_{H(0)} \\ \theta_{H(1)} \\ \vdots \\ \theta_{H(L-1)} \end{bmatrix}, \quad (6)$$

where $\theta_{H(i)} \triangleq \left[vec\left(H(i)\right)^T, vec\left(H(i)^*\right)^T\right]^T \in \mathbb{C}^{2rt\times 1}$. In what follows $k \in \overline{0, L-1}, i \in \overline{1, rt}$. Observe also that $\theta_{\mathcal{H}}^*(2krt+i) = \theta_{\mathcal{H}}\left((2k+1)rt+i\right)$. Let L_b blocks of blind symbols $\mathcal{X}(p), p \in \overline{1, L_b}$ be transmitted. In addition, let the input blind symbol vectors $\mathbf{x}(l), l \in \overline{L_t+1, PL_b+L_t}$ be Gaussian. Then, $\mathcal{R}_{\mathcal{Y}}$, the output correlation matrix is given as $\mathcal{R}_{\mathcal{Y}} \triangleq \mathrm{E}\left\{\mathcal{Y}(l)\mathcal{Y}(l)^H\right\} = \mathcal{H}\mathcal{H}^H + \sigma_n^2\mathbf{I}$, where $\mathcal{R}_y \in \mathbb{C}^{rN\times rN}$. The log-likelihood expression for the semi-blind scenario described above is given as $\mathcal{L}\left(\mathcal{Y};\theta_{\mathcal{H}}\right) = \mathcal{L}_b + \mathcal{L}_t$, where the $\mathcal{L}_b$, the Gaussian log-likelihood of the blind symbols is given as

$$\mathcal{L}_b \triangleq -\sum_{k=1}^{L_b} \mathrm{tr}\left(\mathcal{Y}(k)^H\mathcal{R}_{\mathcal{Y}}^{-1}\mathcal{Y}(k)\right) - L_b \ln\det\mathcal{R}_{\mathcal{Y}}, \quad (7)$$

and $\mathcal{L}_t$, the least-squares log-likelihood of the training part is given as

$$\mathcal{L}_t = \frac{1}{\sigma_n^2}\sum_{i=1}^{L_t}\left\|\mathbf{y}_p(i) - \sum_{j=0}^{L-1} H(j)\mathbf{x}_p(i-j)\right\|^2. \quad (8)$$

Hence, the FIM for the sum likelihood is given as $J_{\theta_{\mathcal{H}}} = J^B + J^t$, where $J^B, J^t \in \mathbb{C}^{2rtL \times 2rtL}$ are the FIMs for the blind and training symbols bursts respectively. Let the block Toeplitz parameter derivative matrix $\mathcal{E}(k) \in \mathbb{C}^{Nr \times (L+N-1)t}$ be defined employing complex derivatives as $\mathcal{E}(krt+i) \triangleq \frac{\partial \mathcal{H}}{\partial \theta_{\mathcal{H}}^{2krt+i}}$. From the results for the Fisher information matrix of a Gaussian process, J^B is given as

$$\begin{aligned}
\frac{J^B_{2krt+i,2lrt+j}}{L_b} &= \left(\frac{J^B_{(2l+1)rt+j,(2k+1)rt+i}}{L_b}\right)^* \\
&= \operatorname{tr}\left(\mathcal{E}(krt+i)\mathcal{H}^H\mathcal{R}_{\mathcal{Y}}^{-1}\mathcal{H}\mathcal{E}(lrt+j)^H\mathcal{R}_{\mathcal{Y}}^{-1}\right) \\
\frac{J^B_{2krt+i,(2l+1)rt+j}}{L_b} &= \left(\frac{J^B_{(2l+1)rt+j,2krt+i}}{L_b}\right)^* \\
&= \operatorname{tr}\left(\mathcal{E}(krt+i)\mathcal{H}^H\mathcal{R}_{\mathcal{Y}}^{-1}\mathcal{E}(lrt+j)\mathcal{H}^H\mathcal{R}_{\mathcal{Y}}^{-1}\right)
\end{aligned}$$

where $J^B_{k,l}$ denotes its $(k,l)^{th}$ element. We have the following result on the rank of the blind FIM for the MIMO FIR channel.

Theorem 1. *The rank of the blind FIM is given as*

$$rank\left(J^B\right) \leq 2rtL - t^2. \tag{9}$$

In fact, if $H(z)$ is irreducible, a basis for the $t^2 \times 1$ nullspace $\mathcal{N}(J_B)$ is given by $U(\mathbf{H})$ as

$$U(\mathbf{H}) \triangleq \left[U(H(0))^T, U(H(1))^T, \ldots, U(H(L))^T\right]^T \tag{10}$$

where the matrix function $U(H): \mathbb{C}^{r \times t} \rightarrow \mathbb{C}^{2rt \times t^2}$ for the matrix $H = [\mathbf{h}_1, \mathbf{h}_2, \ldots, \mathbf{h}_t]$ is defined as

$$U(H) = \begin{bmatrix}
-\mathbf{h}_2^* & \mathbf{0} & -\mathbf{h}_3^* & \ldots & -\mathbf{h}_1^* & \mathbf{0} & \ldots \\
\mathbf{0} & \mathbf{h}_1^* & \mathbf{0} & \ldots & \mathbf{0} & -\mathbf{h}_2^* & \ldots \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \ldots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots \\
\mathbf{0} & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{0} & \ldots \\
\mathbf{0} & \mathbf{h}_2 & \mathbf{0} & \ldots & \mathbf{h}_1 & \mathbf{0} & \ldots \\
\mathbf{h}_1 & \mathbf{0} & \mathbf{0} & \ldots & \mathbf{0} & \mathbf{h}_2 & \ldots \\
\vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \ddots
\end{bmatrix}, \tag{11}$$

Proof. Given in [4]. $\square$

This has significant implications for estimation. As r, L increase, the number of parameters in the system that needs to be identified increases many fold $(2rtL)$ but the number of parameters that cannot be identified from blind data remains fixed at t^2 implying that a wealth of data can be identified without any training. Since the coefficients $H(z)$ come from random fading channels, with high probability, the rank upper bound holds with equality and hence, for improving the clarity of presentation, we drop the '$\leq$' sign and assume that generally the rank upper bound holds with equality.

Recall that $\{\mathbf{x}_p(1), \mathbf{x}_p(2), \ldots, \mathbf{x}_p(L_t)\}$ are the L_t transmitted pilot symbols. Then, the FIM of the training symbols J^t is given as, $J^t = \sum_{i=1}^{L_t} J^t(i)$, where, $J^t(i)$ is the FIM contribution from the i^{th} pilot symbol transmission. Given complex vectors in $\mathbb{C}^{t \times 1}$, let the matrix function $V(i,j)\colon \left(\mathbb{C}^{t\times 1}, \mathbb{C}^{t \times 1}\right) \rightarrow \mathbb{C}^{2rt \times 2rt}$ be defined as

$${}^iV_j \triangleq \begin{bmatrix} \mathbf{x}_p(i)\mathbf{x}_p(j)^H \otimes \mathbf{I}_r & \mathbf{0} \\ \mathbf{0} & \mathbf{x}_p(i)^*\mathbf{x}_p(j)^T \otimes \mathbf{I}_r \end{bmatrix} \tag{12}$$

and ${}^iV_j = \mathbf{0}_{2rt\times 2rt}$ if either i or j is less than or equal to 0. After some manipulations, it can be shown that the FIM contribution $J^t(i) \in \mathbb{C}^{2rtL \times 2rtL}$ is given as

$$J^t(i) = \frac{1}{\sigma_n^2}\begin{bmatrix}
{}^iV_i & {}^iV_{i-1} & \ldots & {}^iV_{i-L+1} \\
{}^{i-1}V_i & {}^{i-1}V_{i-1} & \ldots & {}^{i-1}V_{i-L+1} \\
\vdots & \vdots & \ddots & \vdots \\
{}^{i-L+1}V_i & {}^{i-L+1}V_{i-1} & \ldots & {}^{i-L+1}V_{i-L+1}
\end{bmatrix}. \tag{13}$$

The following result gives the rank of the sum (training + blind) FIM.

Theorem 2. *The rank of the sum (training + blind) FIM $J_{\theta_{\mathcal{H}}}$ is given as*

$$rank\left(J_{\theta_{\mathcal{H}}}\right) = 2rtL - t^2 + \left(2tL_t - {L_t}^2\right), \quad 0 \leq L_t \leq t \tag{14}$$

where L_t is the number of pilot symbols transmitted.

Proof. See Appendix. $\square$

From the above result, one can then obtain a lower bound for the minimum number of training symbols necessary to achieve regularity or a full rank FIM $J_{\theta_{\mathcal{H}}}$. This result is stated below.

Lemma 2. *The number of training symbol transmissions L_t should at least equal the number of transmit antennas t, for the the FIM $J_{\theta_{\mathcal{H}}}$ to be full rank.*

Proof. It is easy to see from (14), that for $L_t = t$, $rank\left(J_{\theta_{\mathcal{H}}}\right) = 2rtL_t$ or full rank. $\square$

Thus by the addition of t pilot symbols, the system becomes completely identifiable which incidentally is also the minimum number of training symbols needed for least-squares pilot based estimation. What then is the advantage of using a semi-blind technique? SB techniques can yield a far lesser MSE of estimation than an exclusively pilot based scheme as illustrated by the following result.

Theorem 3. *Let $J^t = \frac{L_t}{\sigma_n^2}\mathbf{I}_{2rtL}$, which is achieved by an orthogonal pilot sequence as can be seen from* (13)*. Then, as the number of blind symbol transmissions increases ($L_b \to \infty$), the Semi-Blind CRB ${J_{\theta_{\mathcal{H}}}}^{-1}$ approaches the CRB for the exclusive estimation of the t^2 un-constrained parameters,*

$$\lim_{L_b \to \infty} \mathrm{E}\left\{\|H - H\|_F^2\right\} \geq {J_{\theta_{\mathcal{H}}}}^{-1} = \left(\frac{\sigma_n^2}{2L_t}\right) t^2 \tag{15}$$

Proof. Given the fact that $J^t = \frac{L_t}{\sigma_n^2}\mathbf{I}_{2rtL}$, from (**??**), the semi-blind FIM can be expressed as

$$J(\theta_{\mathcal{H}}) = \frac{L_t}{\sigma_n^2}\mathbf{I}_{2rtL\times 2rtL} + \tilde{J}_B, \tag{16}$$

where $J_B\left(\bar{\theta}\right)$ is the blind FIM corresponding to a single observed blind data $\mathcal{Y}$. From theorem 1, $J_B\left(\bar{\theta}\right)$ is rank deficient and in fact $rank\left(J_B\left(\bar{\theta}\right)\right) = 2rt - t^2$. Let the eigen-decomposition of J_B be given as $J_B\left(\bar{\theta}\right) = E_B \Lambda_B {E_B}^H$, where $\Lambda \in \mathcal{D}^{2rt-t^2}$ and $\mathcal{D}$ denotes the space of diagonal matrices. Then,

$$\begin{aligned} J\left(\theta_{\mathcal{H}}\right) &= \frac{L_t}{\sigma_n^2}\left[E_B^\perp, E_B\right]\left[E_B^\perp, E_B\right]^H + E_B\Lambda_B E_B^H \\ &= \left[E_B, E_B^\perp\right]\begin{bmatrix} \frac{L_t}{\sigma_n^2}\mathbf{I} + \Lambda_B & \mathbf{0} \\ \mathbf{0} & \frac{L_t}{\sigma_n^2}\mathbf{I}\end{bmatrix}\left[E_B, E_B^\perp\right]^H \end{aligned}$$

Hence the CRB $J^{-1}\left(\theta_{\mathcal{H}}\right)$ is given as

$$\left[E_B, E_B^\perp\right]\begin{bmatrix} \left(\frac{L_t}{\sigma_n^2}\mathbf{I} + \Lambda_B\right)^{-1} & \mathbf{0} \\ \mathbf{0} & \frac{\sigma_n^2}{L_t}\mathbf{I}\end{bmatrix}\left[E_B, E_B^\perp\right]^H$$

Therefore, as the number of blind symbols $L_b \to \infty \Rightarrow \Lambda_B \to \infty$, the semi-blind bound approaches the constrained bound given as

$$\lim_{N\to\infty} J^{-1}\left(\theta_{\mathcal{H}}\right) = \frac{\sigma_n^2}{L_t} E_B^\perp {E_B^\perp}^H \tag{17}$$

This expression is similar to the one derived in [6]. In fact, the MSE is clearly seen to be given as

$$\begin{aligned} \mathrm{E}\left\{\left\|\hat{\theta}_{\mathcal{H}} - \theta_{\mathcal{H}}\right\|_F^2\right\} &\geq \frac{\sigma_n^2}{L_t}\mathrm{tr}\left(E_B^\perp {E_B^\perp}^H\right) \\ \Rightarrow 2\left(\mathrm{E}\left\{\left\|\hat{H} - H\right\|_F^2\right\}\right) &\geq \frac{\sigma_n^2}{L_t}\mathrm{tr}\left(E_B^\perp {E_B^\perp}^H\right) \\ \mathrm{E}\left\{\left\|\hat{H} - H\right\|_F^2\right\} &\geq \frac{1}{2}\frac{\sigma_n^2}{L_t}\left(2rt - \left(2rt - t^2\right)\right) \\ &= \frac{\sigma_n^2 t^2}{2L_t}, \end{aligned} \tag{18}$$

which is the constrained bound for the estimation of the MIMO channel matrix H. □

Thus the MSE of estimation of the channel matrix $\mathbf{H}$ with the aid of blind information, is directly proportional to t^2 while the MSE of estimation using exclusively pilot symbols is proportional to $2rtL$ the total number of real parameters, given as $\left(\frac{\sigma_n^2}{2L_t}\right) 2rtL$. Hence, the SB MSE is lower by a factor $2\left(\frac{r}{t}\right) L$ and potentially be very efficient compared to blind schemes.

4. SEMI-BLIND ESTIMATION ALGORITHMS

As shown above, the SB problem involves identifying t^2 parameters from the training data. These t^2 parameters correspond to a unitary matrix as illustrated below.

Lemma 3. *Let $H(z) \in \mathbb{C}^{r\times t}(z)$ be the $r \times t$ irreducible channel transfer matrix. Let the input output system model be as shown in (2). Then, $H(z)$ can be identified up to a unitary matrix from the output second order statistics of data.*

The above subspace based result can be found in [7]. From the above result, the matrices $W(i),\ i \in \overline{0, L-1}$ can be estimated blind from the correlation lags $R_y(j),\ j \in \overline{0, L-1}$ and a scheme based on designing multiple delay linear predictors is given in [8] (Set $n_a = 0, d = n_b = L-1$ and it follows that $\tilde{F}_i = W(i)$). It thus remains to compute the unitary matrix $Q \in \mathbb{C}^{t\times t}$ from pilot symbols. Therefore $\mathbf{H} \in \mathbb{C}^{r\times Lt}$ can be written as $\mathbf{HW}\left(\mathbf{I}_L \otimes Q^H\right)$, where $\mathbf{W} \triangleq [W(1), W(2), \ldots, W(L-1)]$. In the next section, we present SB algorithms.

4.1. Orthogonal Pilot ML (OPML) for Q Estimation:

We now describe a procedure to estimate the unitary matrix Q from an orthogonal pilot symbol sequence X_p. Let $X_p(i),\ i \in \overline{0, L-1}$ be defined as

$$X_p(i) \triangleq \left[\mathbf{x}_p(L-i), \mathbf{x}_p(L-i+1), \ldots, \mathbf{x}_p(L_t - i)\right].$$

From (3), $X_p = \left[X_p(0)^T, X_p(1)^T, \ldots, X_p(L-1)^T\right]^T$. The least squares cost function for the constrained estimation of the unitary matrix Q can then be written as

$$\min \left\|Y_p - \sum_{i=0}^{L-1} W(i) Q^H X_p(i)\right\|^2, \quad \text{with} \quad QQ^H = \mathbf{I}_t.$$

Let the pilot matrix X_p be orthogonal, i.e. $X_p X_p^H = L_t\mathbf{I}$. The cost minimizing Q is then given as

$$\hat{Q} = UV^H, \quad \text{where} \quad U\Sigma V^H = \mathrm{SVD}\left(\sum_{i=0}^{L-1} X(i) Y^H W(i)\right).$$

Proof follows from an extension of the result in [5]. Finally, $\hat{\mathbf{H}}$ is given as $\hat{\mathbf{H}} \triangleq \mathbf{W}\hat{Q}^H$. An orthogonal pilot in the context of MIMO FIR channels can be constructed as shown in [9] by employing the Paley Hadamard orthogonal matrix structure.

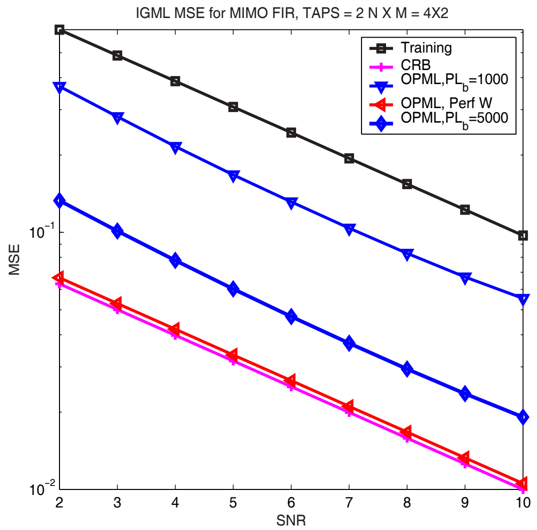

Figure 1: MSE of Estimation of 4×2 with $L = 2$ taps.

5. SIMULATION RESULTS

We consider an $L = 2$ tap, $r = 4, t = 2$ i.e. 4×2 MIMO FIR channel. Each of the elements of $\mathbf{H}$ is generated as a zero-mean circularly symmetric complex Gaussian random variable of unit variance. The orthogonal pilot sequence is constructed by employing a size 20×20 Paley Hadamard matrix. In Fig.1.- we plot the MSE vs SNR when the whitening matrix $W(z)$ is estimated from $PL_b = 1000, 5000$ blind data symbols. For comparison we also plot the CRB given by (15) and also the MSE of estimation with the genie assisted case of perfect knowledge of $W(z)$. The MSE progressively decreases towards the CRB as the number of blind data symbols increases and as illustrated in theorem 3, SB estimation error is $10 \log\left(\frac{32}{4}\right) = 9dB$ lower than the training based scheme.

6. CONCLUSION

In this work we have investigated the rank properties of the FIM of a MIMO FIR channel and demonstrated that at least t pilot symbol transmissions are necessary to achieve a full rank FIM for an L tap $r \times t$ ($r \geq t$) channel. An irreducible channel transfer function $H(z)$ can be decomposed as $H(z) = W(z)Q^H$, where $W(z)$ can be estimated from the blind data alone. Constrained estimation schemes have been presented to estimate the unitary matrix Q from pilot symbols. Simulation results demonstrate the performance of the proposed scheme.

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