A SEMI-BLIND MIMO CHANNEL ESTIMATION SCHEME FOR MRT

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ABSTRACT

In this paper, we investigate semi-blind channel estimation for multiple input multiple output (MIMO) quasi-static flat fading channels when maximum ratio transmission (MRT) is employed. We propose a closed-form semi-blind solution (CFSB) for estimating the optimum transmit and receive beamforming vectors of the channel matrix. Employing matrix perturbation theory, we develop expressions for the mean squared error (MSE) in the beamforming vector and average received SNR of both the semi-blind and the conventional least squares estimation (CLSE) schemes. It is found that the proposed estimation technique outperforms CLSE for a wide range of training lengths and training SNRs.

I. INTRODUCTION

Semi-blind techniques can enhance the accuracy of channel estimation by efficiently utilizing not only the known training symbols but also the unknown data symbols [1, 2]. However, the previous techniques for channel estimation in MIMO systems are transmission scheme agnostic, and typically assume that the transmitted data is spatially white. This requirement poses new challenges in channel estimation for feedback based schemes, as they do not posses the spatial whiteness property. Also, accurate channel estimation takes on additional importance for feedback based communication schemes as the quality of feedback has significant implications on their performance. On the other hand, feedback based transmission schemes may allow greater estimation accuracy for a given level of training, as they require estimation of fewer channel parameters than transmission schemes without feedback. For example, channel estimation algorithms when MRT is employed at the transmitter only need to estimate the transmit and receive beamforming vectors $\mathbf{v}_1$ and $\mathbf{u}_1$, the right and left dominant singular vectors of H respectively [3], where H is the $r \times t$ channel transfer matrix, and $r\,/\,t$ are the number of receive / transmit antennas. In this study, we consider semi-blind estimation algorithms specifically designed for beamforming-based MIMO communication, and look at some of the issues alluded to above. The contributions of this paper are as follows:

- We describe the training based conventional least squares estimation (CLSE) algorithm, and analytically compute its performance in terms of the mean squared error (MSE) in the estimated beamforming vector $\mathbf{v}_1$ and the channel gain.
- We propose an alternative closed form semi-blind (CFSB) algorithm that estimates $\mathbf{u}_1$ from the data using a blind algorithm, and uses the training symbols exclusively to estimate $\mathbf{v}_1$.
- We motivate and introduce a new signal transmission scheme that enables implementation of the CFSB scheme.
- We derive theoretical expressions for the performance of the proposed CFSB algorithm and compare it to the CLSE scheme.

We validate the analytical results on the performance of these two techniques through simulations.

II. SYSTEM MODEL AND NOTATION

The MIMO channel input-output equation at time k is

$$\mathbf{y}_k = H\mathbf{x}_k + \mathbf{n}_k, \tag{1}$$

where $\mathbf{y}_k \in \mathbb{C}^r$ is the channel output, $\mathbf{x}_k \in \mathbb{C}^t$ is the channel input, and $\mathbf{n}_k$ is additive white Gaussian noise with zero mean and covariance matrix I_r, the $r \times r$ identity matrix. The channel transfer matrix $H \in \mathbb{C}^{r\times t}$ is assumed to be quasi-static flat-fading. Let the singular value decomposition (SVD) of H be given as $H = U\Sigma V^H$, and $\Sigma \in \mathbb{R}^{r\times t}$ contains singular values $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_m > 0$, along the diagonal, where $m = \operatorname{rank}(H)$. The training symbols are stacked together to form a training symbol matrix $X_p \in \mathbb{C}^{t\times L}$ as $X_p = [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_L]$. To simplify analysis, we assume that orthogonal training sequences are used, that is, $X_pX_p^H = \gamma_p I_t$, where $\gamma_p \triangleq LP_T/t$. The data symbols $\mathbf{x}_k$ could either be spatially-white (i.e., $\mathbf{E}\left\{\mathbf{x}_k\mathbf{x}_k^H\right\} = (P_D/t)\, I_t$), or it could be the result of using beamforming at the transmitter with unit-norm weight vector $\mathbf{w} \in \mathbb{C}^{t\times 1}$ $\left(\text{i.e., } \mathbf{E}\{\mathbf{x}_k\mathbf{x}_k^H\} = P_D\mathbf{w}\mathbf{w}^H\right)$, where the data transmit power is $\mathbf{E}\left\{\mathbf{x}_k^H\mathbf{x}_k\right\} = P_D$. We let $N(> L)$ denote the number of spatially-white data symbols transmitted, that is, a total of $N + L$ symbols are transmitted prior to transmitting beamformed-data. Note that the N white data symbols carry (unknown) information bits, and hence are not a waste of available bandwidth.

In this paper, we restrict our attention to the case where the transmitter employs MRT to send data, that is, a *single* data stream is transmitted over t transmit antennas after passing through a beamformer $\mathbf{w}$. Given the channel matrix H, the optimum choice of $\mathbf{w}$ is $\mathbf{v}_1$ [3]. Thus, MRT only needs an accurate estimate of $\mathbf{v}_1$ to be fed-back to the transmitter. We assume that $t \geq 2$, since when $t = 1$, estimation of the beamforming vector has no relevance. Finally, we will compare the performance of different

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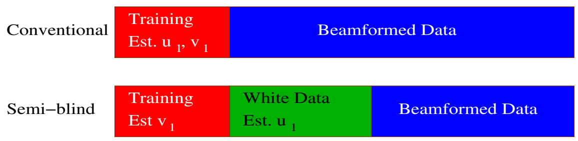

Fig. 1. Comparison of the transmission scheme for conventional least-squares (CLSE) and closed-form semi-blind (CFSB) estimation.

estimation techniques using two measures, the MSE in the estimate of $\mathbf{v}_1$ and the gain (the power amplification/attenuation) of the one-dimensional channel resulting from beamforming with the estimated vector $\hat{\mathbf{v}}_1$.

III. CONVENTIONAL LEAST SQUARES ESTIMATION

For an orthogonal training sequence X_p, the least-squares ML estimate of the channel matrix $\hat{H}_c$, is first obtained as

$$\hat{H}_c = Y_p X_p^{\dagger} = \frac{1}{\gamma_p} Y_p X_p^H, \tag{2}$$

where $X_p^{\dagger}$ is the pseudo-inverse of X_p, and $Y_p\,(= HX_p + \eta_p)$ is the set of received training symbol vectors. By the invariance property of ML estimators, the ML estimate of $\mathbf{v}_1$ and $\mathbf{u}_1$, denoted $\hat{\mathbf{v}}_c$ and $\hat{\mathbf{u}}_c$ respectively, is now obtained via a SVD of $\hat{H}_c$, the ML estimate of the channel matrix H.

IV. SEMI-BLIND ESTIMATION (CFSB)

If the transmitted data symbols are spatially-white, the ML estimate of $\mathbf{u}_1$ can be obtained from the entire received symbols by computing the following SVD

$$\hat{U}\hat{\Sigma}^2\hat{U}^H = \frac{1}{\gamma_d}\left(\hat{R}_y - NI_r\right), \tag{3}$$

where $\hat{R}_y \triangleq \sum_{i=1}^{N} \mathbf{y}_i \mathbf{y}_i^H$ and $\gamma_d \triangleq NP_D/t$ is the transmit data SNR, per antenna. The estimate of $\mathbf{u}_1$, denoted $\hat{\mathbf{u}}_s$, is thus computed blind from the received data as the first column of $\hat{U}$. The MMSE estimate of $\mathbf{v}_1$ (under $\|\mathbf{v}_1\| = 1$) with perfect knowledge of $\mathbf{u}_1$ is then given by the solution to the following constrained least squares problem:

$$\hat{\mathbf{v}}_s = \arg \min_{\mathbf{v} \in \mathbb{C}^t,\ \|\mathbf{v}\|=1} \|\tilde{Y}_p - \mathbf{v}^H \tilde{X}_p\|^2, \tag{4}$$

where $\tilde{Y}_p \triangleq \frac{\mathbf{u}_1{}^H Y_p}{\sigma_1 \gamma_p}$, and $\tilde{X}_p \triangleq \frac{X_p}{\gamma_p}$. The unit-norm constrained estimate $\hat{\mathbf{v}}_s$ which minimizes the above cost-function can then be shown [4] to be given as,

$$\hat{\mathbf{v}}_s = \frac{X_p Y_p^H \mathbf{u}_1}{\|X_p Y_p^H \mathbf{u}_1\|}. \tag{5}$$

We estimate $\mathbf{v}_1$ from the training symbols X_p, by substituting $\hat{\mathbf{u}}_s$ for $\mathbf{u}_1$ in (5). Fig. 1 shows a schematic representation of the CLSE and the CFSB schemes.

MRC Receive Beamforming: An alternative to employing $\hat{\mathbf{u}}_1$ at the receiver is to use an estimate of the maximum ratio combining (MRC) beamforming vector $H\hat{\mathbf{v}}_1/\|H\hat{\mathbf{v}}_1\|$, denoted as $\hat{\mathbf{u}}_1'$ which can be accurately estimated *blind* by computing the dominant eigenvector of the output covariance $\hat{R}_y$ of the received *beamformed* data. We present the theoretical analysis of this technique as well.

V. MSE AND SNR ANALYSIS

We recapitulate a result from matrix perturbation theory that is extensively used in the analysis [5]. Consider a first order perturbation of a hermitian symmetric matrix R by an error matrix ΔR to get $\hat{R}$, that is, $\hat{R} = R + \Delta R$. Then, if the eigenvalues of R are distinct, for small perturbations, the eigenvectors $\hat{\mathbf{s}}_k$ of $\hat{R}$ can be approximately expressed in terms of the eigenvectors $\mathbf{s}_k$ of R as

$$\hat{\mathbf{s}}_k = \mathbf{s}_k + \sum_{\substack{r=1\\ r\neq k}}^{L} \frac{\mathbf{s}_r^H \Delta R \mathbf{s}_k}{\lambda_k - \lambda_r} \mathbf{s}_r, \tag{6}$$

where L is the rank of R, λ_k is its k-th eigenvalue, and $\lambda_k \neq \lambda_j,\ k \neq j$. When $k = 1$, $\tilde{\mathbf{s}}_1 = \hat{\mathbf{s}}_1/\|\hat{\mathbf{s}}_1\| = S\mathbf{d}$, where $\mathbf{d} = [1+\Delta d_1, \Delta d_2, \ldots, \Delta d_n]^T$. If Δd_i are small, since $\|\mathbf{d}\| = 1$, the components Δd_i can be approximately expressed as

$$\begin{aligned} \Delta d_i &\simeq \frac{\mathbf{s}_i^H \Delta R \mathbf{s}_1}{\lambda_1 - \lambda_i},\ i = 2, \ldots, n \\ \Delta d_1 &\simeq -\frac{1}{2}\sum_{i=2}^{t} |\Delta d_i|^2 . \end{aligned} \tag{7}$$

Note that Δd_1 is real, and is a second-order term compared to $\Delta d_i,\ i \geq 2$. We will use this fact in our first-order approximations to ignore terms such as $|\Delta d_1|^2, |\Delta d_1|^3, \ldots$ and $|\Delta d_i|^3, |\Delta d_i|^4, \ldots,$ $i \geq 2$. In the sequel, we assume that the dominant singular value of H is distinct, so the conditions required for the above result are valid. For the sake of brevity, we provide the detailed derivation only for the MSE in $\hat{\mathbf{v}}_c$ and the channel gain with MRC for the CLSE scheme. The other results can be derived by following a similar procedure; interested readers may also refer to [4] for more details.

A. Conventional Least-Squares Estimation (CLSE)

A.1. MSE in $\hat{\mathbf{v}}_c$ for the CLSE scheme

To compute the MSE in $\hat{\mathbf{v}}_c$, we use (2), to write the matrix $\hat{H}_c^H \hat{H}_c$ as a perturbation of $H^H H$ as,

$$\hat{H}_c^H \hat{H}_c \simeq V\Sigma^2 V^H + E_t, \tag{8}$$

where $E_t = \left[V\Sigma U^H E_p + E_p{}^H U \Sigma V^H\right]$ and the matrix $E_p \triangleq \frac{1}{\gamma_p}\eta_p X_p^H$. Recall that $\hat{\mathbf{v}}_c$ is estimated from the SVD of $\hat{H}_c$. We can let $\hat{\mathbf{v}}_c = V\mathbf{d}$, and write $\mathbf{d} = [1+\Delta d_1, \Delta d_2, \ldots, \Delta d_t]^T$ as a perturbation of $[1, 0, \ldots, 0]^T$. For $i \geq 2$, Δd_i is obtained from (7) as

$$\Delta d_i = \frac{\mathbf{v}_i{}^H E_t \mathbf{v}_1}{\sigma_1^2 - \sigma_i^2} = \frac{\sigma_i \mathbf{u}_i^H E_p \mathbf{v}_1 + \sigma_1 \mathbf{v}_i^H E_p{}^H \mathbf{u}_1}{\sigma_1^2 - \sigma_i^2}. \tag{9}$$

The following result is used to find $\mathbf{E}\left\{|\Delta d_i|^2\right\}$.

Lemma 1 *Let $\mu_1, \mu_2 \in \mathbb{C}$ be fixed complex numbers. Let $\sigma_p^2 = \frac{1}{\gamma_p}$ denote the variance of one of the elements of E_p. Then, $\Omega \triangleq \mathbf{E}\left\{\left|\mu_1 \mathbf{u}_i{}^H E_p \mathbf{v}_j + \mu_2 \mathbf{v}_i{}^H E_p{}^H \mathbf{u}_j\right|^2\right\} = \sigma_p^2\left(|\mu_1|^2 + |\mu_2|^2\right)$, for any $1 \leq i \leq r,\ 1 \leq j \leq t$.*

Proof: Let $a \triangleq \mathbf{u}_i{}^H E_p \mathbf{v}_j$ and $b \triangleq \mathbf{v}_i{}^H E_p{}^H \mathbf{u}_j$. Then, a and b are circularly symmetric random variables. Since E_p is circularly symmetric ($\mathbf{E}\{E_p(i,j)\, E_p(k,l)\} = 0, \forall\, i,j,k,l$) and a and b^* are both linear combinations of elements of E_p, we have $\mathbf{E}\{ab^*\} = 0$. Finally, since $\|\mathbf{u}_i\| = \|\mathbf{v}_j\| = 1$, the variance of a and b are equal, and $\sigma_a^2 = \sigma_b^2 = \sigma_p^2$. Substituting, we have

$$\begin{aligned} \Omega &= |\mu_1|^2\, \sigma_a^2 + |\mu_2|^2\, \sigma_b^2 \\ &= \sigma_p^2(|\mu_1|^2 + |\mu_2|^2). \end{aligned}$$

Which concludes the proof. ■

Using the above lemma with $\mu_1 = \sigma_i, \mu_2 = \sigma_1$ and $j = 1$, we get, for $i \geq 2$,

$$\mathbf{E}\left\{|\Delta d_i|^2\right\} = \sigma_p^2 \frac{\sigma_1^2 + \sigma_i^2}{(\sigma_1^2 - \sigma_i^2)^2}, \tag{10}$$

where the expectation is taken with respect to the AWGN term η_p. The following lemma helps simplify the expression further. We omit the proof, as it is straightforward.

Lemma 2 *If $\hat{\mathbf{v}}_c = V\mathbf{d}$, then*

$$\|\hat{\mathbf{v}}_c - \mathbf{v}_1\|^2 = 2\left(1 - \mathrm{Re}(d_1)\right) = -\left(\Delta d_1 + \Delta d_1^*\right), \tag{11}$$

where $d_1 = 1 + \Delta d_1$ is the first element of $\mathbf{d}$.

Using (10) in (7) and substituting into in (11), the final expression for the MSE in $\hat{\mathbf{v}}_c$ for the CLSE is

$$\mathbf{E}\left\{\|\hat{\mathbf{v}}_c - \mathbf{v}_1\|^2\right\} = \frac{1}{\gamma_p}\sum_{i=2}^{t} \frac{\sigma_1^2 + \sigma_i^2}{(\sigma_1^2 - \sigma_i^2)^2}. \tag{12}$$

A.2. Channel Gain with $\hat{\mathbf{u}}_c$ for the CLSE scheme

The channel gain when using $\hat{\mathbf{u}}_c$ and $\hat{\mathbf{v}}_c$ as beamforming vectors at the receiver and the transmitter respectively, given by $\rho_c = \mathbf{E}\left\{\left|\hat{\mathbf{u}}_c^H H \hat{\mathbf{v}}_c\right|^2\right\}$ can be shown to be

$$\sigma_1^2 - \frac{1}{\gamma_p}\left(r + t - 2\cdot\mathrm{rank}(H)\right) - \frac{2}{\gamma_p}\sum_{i=2}^{\mathrm{rank}(H)} \frac{\sigma_1^2}{\sigma_1^2 - \sigma_i^2}. \tag{13}$$

Note that $\rho_c \leq \rho_p \triangleq \sigma_1^2$, which is the gain with perfect beamforming at both the transmitter and the receiver. As $\gamma_p = LP_T/t$ increases, ρ_c approaches ρ_p. Note that, when $r = 1$, the above expression simplifies to $\rho_c = \rho_p - \frac{1}{\gamma_p}(t-1)$. Also, when $r = t$, the CLSE performs best when the channel is spatially single dimensional, that is, $\sigma_i = 0,\ i \geq 2$. In this case, we have $\rho_c = \rho_p - \frac{2}{\gamma_p}(t-1)$.

A.3. Channel Gain with MRC for the CLSE scheme

When $\hat{\mathbf{u}}_c'$ is used as the receive beamforming vector, the channel gain $\rho_s \triangleq \mathbf{E}\left\{\hat{\mathbf{v}}_c^H H^H H \hat{\mathbf{v}}_c\right\}$ is given by

$$\begin{aligned} \rho_s &= \mathbf{E}\left\{\hat{\mathbf{v}}_c^H H^H H \hat{\mathbf{v}}_c\right\} = \mathbf{E}\left\{\mathbf{d}^H \Sigma^2 \mathbf{d}\right\}, \\ &= \sigma_1^2 - \sum_{i=2}^{t}\left(\sigma_1^2 - \sigma_i^2\right)\mathbf{E}\left\{|\Delta d_i|^2\right\}. \end{aligned}$$

Substituting for $\mathbf{E}\left\{|\Delta d_i|^2\right\}$ from (10), we get

$$\rho_s = \sigma_1^2 - \frac{1}{\gamma_p}\sum_{i=2}^{t}\frac{\sigma_1^2 + \sigma_i^2}{\sigma_1^2 - \sigma_i^2} \tag{14}$$

Comparing, it can be seen that (14) outperforms (13), since $\sigma_1 > \sigma_i,\ i \geq 2$.

B. Closed Form Semi-Blind (CFSB) Estimation

B.1. MSE in $\hat{\mathbf{v}}_s$ of the CFSB Scheme

Let $\gamma_p = LP_T/t$ and $\gamma_d = NP_D/t$. The error in $\hat{\mathbf{v}}_s$ can be thought of as the sum of two terms: the first one being the error due to the noise in the white (unknown) data, and the second being the error due to the noise in the training data. A similar decomposition can be used to express the loss in channel gain (relative to σ_1^2). Using a high-SNR first order perturbation analysis, the contributions of these two components can be computed and the final expressions for the MSE in $\hat{\mathbf{v}}_s$ can be derived as,

$$\frac{(2t-1)}{2\gamma_p\sigma_1^2} + \sum_{i=2}^{r}\frac{\sigma_i^2}{\sigma_1^2\left(\sigma_1^2 - \sigma_i^2\right)^2}\left(\frac{\sigma_1^2\sigma_i^2}{N} + \frac{\sigma_i^2 + \sigma_1^2}{\gamma_d} + \frac{N}{\gamma_d^2}\right).$$

An interesting relation between the above expression and the Cramer-Rao lower bound (CRB) for the constrained estimation of $\mathbf{v}_1$ is stated in the following theorem.

Theorem 1 *The error component $\frac{(2t-1)}{2\gamma_p\sigma_1^2}$ is the CRB for the constrained estimation of $\mathbf{v}_1$ under perfect knowledge of $\mathbf{u}_1$, that is,*

$$\mathbf{E}\left\{\|\mathbf{v}_1 - \hat{\mathbf{v}}_1\|^2\right\} \geq \frac{(2t-1)}{2\gamma_p\sigma_1^2}. \tag{15}$$

Proof: Since $\tilde{Y_p} = \mathbf{v}_1{}^H\tilde{X_p} + \tilde{n}$, the effective SNR for estimation of $\mathbf{v}_1$ is SNR $\gamma_s = \gamma_p\sigma_1^2$. From the results derived for the CRB with constrained parameters [6], since $\tilde{X_p}\tilde{X_p}^H = I_t/\gamma_p$, the estimation error in $\mathbf{v}_1$ is proportional to the number of parameters, which equals $2t-1$ as $\mathbf{v}_1$ is a t-dimensional complex vector with one constraint ($\|\mathbf{v}_1\| = 1$). The estimation error is given by

$$\mathbf{E}\left\{\|\hat{\mathbf{v}}_s - \mathbf{v}_1\|^2\right\} \geq \frac{\{\text{Num. Parameters}\}}{2\gamma_s} = \frac{(2t-1)}{2\gamma_p\sigma_1^2},$$

which proves the desired result. ■

B.2. Channel Gain with $\hat{\mathbf{u}}_s$ of the CFSB Scheme

The power amplification $\rho_u \triangleq \mathbf{E}\left\{\left|\mathbf{u}_1{}^H H\hat{\mathbf{v}}_s\right|^2\right\}$ conditioned on the channel matrix H, can be shown to be given as,

$$\sigma_1^2 - \frac{t-1}{\gamma_p} - \sum_{i=2}^{r}\frac{1}{(\sigma_1^2 - \sigma_i^2)}\left(\frac{\sigma_1^2\sigma_i^2}{N} + \frac{\sigma_1^2 + \sigma_i^2}{\gamma_d} + \frac{N}{\gamma_d^2}\right).$$

Comparing the above expression with the power amplification of CLSE (13), we see that when N is large (perfect $\mathbf{u}_1$) and $r = t$, even in the best case of a spatially single-dimensional channel, $\rho_c = \rho_p - \frac{2}{\gamma_p}(t-1) < \rho_u$. Next, when $r = 1$, CLSE and CFSB techniques perform exactly the same: $\rho_c = \rho_u = \sigma_1^2 - \frac{t-1}{\gamma_p}$ since $\mathbf{u}_1 = 1$ (that is, no receive beamforming is needed). Thus, if perfect knowledge of $\mathbf{u}_1$ is available at the receiver, CFSB is guaranteed to perform as well as CLSE, regardless of the training symbol SNR. It has been observed through simulations that the CFSB outperforms the CLSE scheme in many cases for reasonable values of N.

B.3. Channel Gain with MRC of the CFSB Scheme

When MRC is employed at the receiver,

$$\rho = \sigma_1^2 - \frac{t-1}{\gamma_p} + \frac{1}{\gamma_p}\sum_{i=2}^{t}\frac{\sigma_i^2}{\sigma_1^2} - \sum_{i=2}^{r}\frac{\sigma_i^2}{\sigma_1^2(\sigma_1^2-\sigma_i^2)} \times \left(\frac{\sigma_1^2\sigma_i^2}{N} + \frac{\sigma_i^2+\sigma_1^2}{\gamma_d} + \frac{N}{\gamma_d^2}\right)$$

Comparing the gains, it can be seen that the channel gain with MRC is higher than that with using the estimate $\hat{\mathbf{u}}_s$ as the receive beamforming vector.

VI. SIMULATION RESULTS

The simulation setup consists of a Rayleigh flat fading channel with 4 transmit antennas and 4 receive antennas ($t = r = 4$). The data (and training) are drawn from a 16-QAM constellation. 10,000 random instantiations of the channel were used in the averaging. The results of our simulations are presented in the following graphs.

Experiment 1: Fig.2 shows the MSE performance of the CLSE- and the CFSB schemes at two different values of N, as well as the $N = \infty$ (perfect knowledge of U) case. For comparison, we also plot the performance of the so-called orthogonal pilot maximum likelihood (OPML) technique, presented in [6], which uses the white data to estimate the entire U matrix, and uses the training to estimate V. At $N = 100$ white data symbols, the CLSE technique outperforms the CFSB for $L \geq 20$, as the error in $\mathbf{u}_1$ dominates the error in the semi-blind technique. As white data length increases, the CFSB performs progressively better than the CLSE. Also, in the presence of a finite number (N) of white data, the CFSB outperforms the OPML scheme as CFSB only requires an accurate estimate of the dominant eigenvector $\mathbf{u}_1$ from the white data, which is easier to obtain compared to the entire U matrix.

Experiment 2: Next, as an example of overall performance comparison, fig.3 shows the BER performance versus the data SNR of the different estimation schemes for a 2×2 system, with uncoded 4-QAM transmission, $L = 2$ training symbols, $N = 16$ white data symbols (for the semi-blind technique) and a frame size $L_d = 500$ symbols. The CFSB scheme outperforms the CLSE scheme in terms of its BER performance, including the effect of white data transmission.

VII. CONCLUSION

We have investigated semi-blind channel estimation for MIMO flat-fading channels with MRT, in terms of the MSE in the beamforming vector $\mathbf{v}_1$ and received SNR. The CFSB scheme is proposed as a closed-form semi-blind solution for estimating the optimum transmit beamforming vector $\mathbf{v}_1$. Analytical expressions for the mean-squared error (MSE) and the channel power gain of both the CLSE and the CFSB estimation schemes are developed, which can be used to theoretically compare their performance. Simulations illustrate the relative performance of the different techniques.

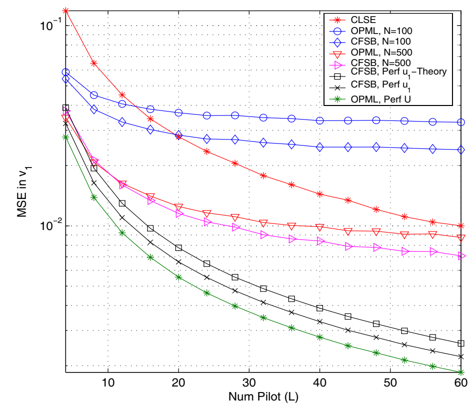

Fig. 2. MSE in $\mathbf{v}_1$ vs L, 4×4 system, for two different values of N, and data and training symbol SNR fixed at $P_T = P_D = 10$dB. The two competing semi-blind techniques, OPML and CFSB, are plotted. For finite N, CFSB outperforms OPML as it only requires an accurate estimate of $\mathbf{u}_1$ from the blind data.

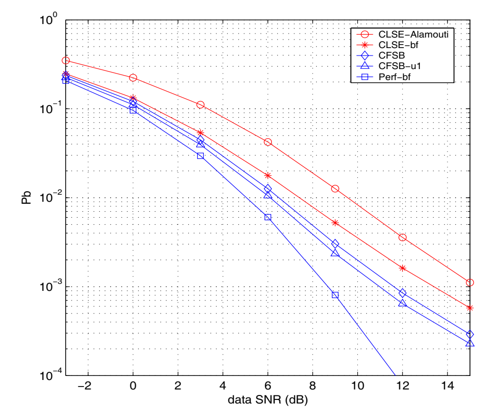

Fig. 3. Bit error probability versus data SNR for the 2×2 system, with $L = 2, N = 16, \gamma_p = 2dB$

VIII. REFERENCES

[1] D. Pal, "Fractionally spaced equalization of multipath channels: a semi-blind approach," *1993 International Conference on Acoustics, Speech and Signal Processing*, vol. 3, pp. 9–12, 1993.

[2] E. De Carvalho and D. T. M. Slock, "Asymptotic performance of ML methods for semi-blind channel estimation," *Thirty-First Asilomar Conference*, vol. 2, pp. 1624–8, 1998.

[3] T. K. Y. Lo, "Maximum ratio transmission," *IEEE Trans. Commun.*, vol. 47, no. 10, pp. 1458–1461, Oct. 1999.

[4] C. R. Murthy, A. K. Jagannatham, and B. D. Rao, "Semi-blind MIMO channel estimation for maximum ratio transmission," *submitted to IEEE Transactions on Wireless Communications*.

[5] J. H. Wilkinson, *The Algebraic Eigenvalue Problem*, Oxford University Press, Walton St., Oxford, first edition, 1965.

[6] A. K. Jagannatham and B. D. Rao, "Whitening rotation based semi-blind MIMO channel estimation," *IEEE Trans. on Sig. Proc.*, Submitted.