Robust Semi-Blind Estimation for Beamforming Based MIMO Wireless Communication

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***Abstract*— In this paper, we present robust semi-blind (SB) algorithms for the estimation of beamforming vectors for multiple-input multiple-output wireless communication. The transmitted symbol block is assumed to comprise of a known sequence of training (pilot) symbols followed by information bearing blind (unknown) data symbols. Analytical expressions are derived for the robust SB estimators of the MIMO receive and transmit beamforming vectors. These robust SB estimators employ a preliminary estimate obtained from the pilot symbol sequence and leverage the second-order statistical information from the blind data symbols. We employ the theory of Lagrangian duality to derive the robust estimate of the receive beamforming vector by maximizing an inner product, while constraining the channel estimate to lie in a confidence sphere centered at the initial pilot estimate. Two different schemes are then proposed for computing the robust estimate of the MIMO transmit beamforming vector. Simulation results presented in the end illustrate the superior performance of the robust SB estimators.**

I. Introduction

Multiple-input multiple-output (MIMO) communication has received significant attention over the past decade due to its promise of higher capacity from spatial multiplexing and resilience to channel fading due to its diversity advantage. Accurate channel estimation is key to realizing many of the gains in practical MIMO systems. However, previous techniques for channel estimation in MIMO systems are not optimized for the underlying transmission scheme. For instance, many schemes typically estimate the entire MIMO channel matrix H, where H is the $r \times t$ channel transfer matrix, and $r\,/\,t$ are the number of receive / transmit antennas. However, when a scheme such as maximum-ratio transmission (MRT) is employed at the transmitter, the receiver channel estimation algorithms only need to estimate the transmit and receive beamforming vectors $\mathbf{v}_1$ and $\mathbf{u}_1$, the right and left dominant singular vectors of H respectively [1], and not the entire channel matrix H. This is in contrast to open-loop schemes such as the ones employed in space-time coding based receivers, where it is necessary to estimate the entire channel matrix H in order to decode and detect the data. Thus, feedback based communication systems may allow greater estimation accuracy for a given level of training, as they require estimation of fewer channel parameters than transmission schemes without feedback.

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Further, accurate channel estimation is especially important for feedback based systems, as the accuracy of the fedback vector estimate has significant implications on the performance of such systems. This requirement poses new challenges in channel estimation for feedback based MIMO transmission schemes. Conventional estimation schemes rely on the transmission of a known sequence of training symbols, also known as pilot symbols, to estimate the channel. Semi-blind techniques [2], [3] can enhance the accuracy of the channel estimate by efficiently utilizing not only the known training symbols but also the unknown data symbols. In [4], [5], [6], an orthogonal pilot based maximum likelihood (OPML) semi-blind estimation scheme is proposed, where the channel matrix H is factored into the product of a whitening matrix W and a unitary rotation matrix Q. W is estimated from the data using a blind algorithm, while Q is estimated exclusively from the training data using the OPML algorithm. A semi-blind scheme for the estimation of the MIMO beamforming vectors was introduced in [7]. While this method very frequently yields estimates with greater estimation accuracy than the conventional (pilots-only) based scheme, its estimation accuracy is critically dependent on the accuracy of the blind estimate, and as a result, the semi-blind estimate sometimes has a lower accuracy than the training-based estimate.

To overcome this problem, in this study, we consider robust semi-blind estimation algorithms specifically designed for beamforming-based MIMO communication. In this technique, we initially compute a rudimentary estimate of the desired parameter vector from a sequence of training symbols. We then form a robust estimate by employing the statistical information available from the blind symbols, thus enhancing the accuracy of the initial estimate. In spirit, this follows the doubly-constrained robust Capon beamformers proposed in [8]. The robust estimate of the receive beamforming vector $\mathbf{u}_1$ is derived as the solution of a Lagrangian dual optimization problem, which constrains the estimate to lie in a ball centered at the preliminary estimate. Following this, two different schemes are proposed for obtaining a robust estimate of the transmit beamforming vector $\mathbf{v}_1$. Simulations results are presented to illustrate the improved performance achievable by robust semi-

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blind estimation.

The rest of this paper is organized as follows. In Section II, we introduce the system model and notation. In Sections III and IV, we derive the robust semi-blind estimation algorithms for the receive and transmit beamforming vectors, respectively. Simulation results are presented in V. Finally, we offer our conclusions in VI.

II. System model and notation

Consider a MIMO channel with input-output equation at time k given by

$$\mathbf{y}_k = H\mathbf{x}_k + \mathbf{n}_k, \tag{1}$$

where $\mathbf{y}_k \in \mathbb{C}^r$ is the channel output, $\mathbf{x}_k \in \mathbb{C}^t$ is the channel input, and $\mathbf{n}_k$ is additive white Gaussian noise with zero mean and covariance matrix I_r, the $r \times r$ identity matrix. The channel transfer matrix $H \in \mathbb{C}^{r\times t}$ is assumed to be quasi-static flat-fading. Let the singular value decomposition (SVD) of H be given as $H = U\Sigma V^H$, and $\Sigma \in \mathbb{R}^{r\times t}$ contains singular values $\sigma_1 \geq \sigma_2 \geq \ldots \geq \sigma_m > 0$, along the diagonal, where $m = \mathrm{rank}(H)$. The unitary matrices U and V have the right and left singular vectors of H as their columns, respectively. Let $\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_L$ be the training symbols transmitted for the purposes of channel estimation, and let the $t \times L$ matrix X_p be defined by stacking the training symbols as $X_p \triangleq [\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_L]$. To simplify analysis, we assume that orthogonal training sequences are used, that is, $X_p X_p^H = \gamma_p I_t$, where $\gamma_p \triangleq LP_T/t$. The data symbols $\mathbf{x}_k$ could either be spatially-white (i.e., $\mathbf{E}\left\{\mathbf{x}_k\mathbf{x}_k^H\right\} = (P_D/t)\, I_t$), or could be the result of using beamforming at the transmitter with a unit-norm weight vector $\mathbf{w} \in \mathbb{C}^{t\times 1}$ $\left(\text{i.e., } \mathbf{E}\{\mathbf{x}_k\mathbf{x}_k^H\} = P_D\mathbf{w}\mathbf{w}^H\right)$, where the data transmit power is $\mathbf{E}\left\{\mathbf{x}_k^H\mathbf{x}_k\right\} = P_D$. We let $N(> L)$ denote the number of spatially-white data symbols transmitted, that is, a total of $N + L$ symbols are transmitted prior to transmitting beamformed-data. Note that the N white data symbols carry (unknown) information bits, and hence are not a waste of available bandwidth.

In this paper, we restrict our attention to the case where the transmitter employs maximum ratio transmission (MRT) to send beamformed-data, that is, a *single* data stream is transmitted over t transmit antennas after passing through a beamformer $\mathbf{w}$. Given the channel matrix H, the optimum choice of $\mathbf{w}$ is $\mathbf{v}_1$ [1]. Thus, MRT only needs an accurate estimate of $\mathbf{v}_1$ to be fed-back to the transmitter. We assume that $t \geq 2$, since when $t = 1$, estimation of the beamforming vector has no relevance. We also assume that the receiver employs maximum ratio combining (MRC), i.e., the received vector is filtered by the receive beamforming vector followed by detection of the transmitted symbols. When the transmitter employs MRT, the MRC beamforming vector at the receiver is given by $\mathbf{u}_1$. Finally, we will compare the performance of different estimation techniques using two measures, the MSE in the estimate of the beamforming vector and the gain (the power amplification/attenuation) of the one-dimensional channel resulting from beamforming with the estimated vector.

III. Receive Beamforming Estimation

A. Conventional (Pilots only) Estimation

In this section, we will find the robust estimate of the receive beamforming vector. We start by describing the conventional least-squares estimation (CLSE) scheme. When an orthogonal training sequence X_p is employed, the least-squares estimate of the channel matrix $\hat{H}_c$, is first obtained as

$$\hat{H}_c = Y_p X_p^{\dagger} = \frac{1}{\gamma_p} Y_p X_p^H, \tag{2}$$

where $X_p^{\dagger}$ is the pseudo-inverse of X_p, and Y_p given by the relation $(HX_p + \eta_p)$ is the set of received training symbol vectors. Since the noise is assumed white, $\hat{H}_c$ is also the ML estimate of H. By the invariance property of ML estimators [9], the ML estimate of $\mathbf{v}_1$ and $\mathbf{u}_1$, denoted $\hat{\mathbf{v}}_c$ and $\hat{\mathbf{u}}_c$ respectively, is now obtained via a SVD of $\hat{H}_c$, the ML estimate of the channel matrix H. These quantities $\hat{\mathbf{u}}_c, \hat{\mathbf{v}}_c$ represent the conventional estimates of the receive and transmit beamforming vectors respectively.

B. Robust Semi-Blind Estimation

We now describe the robust semi-blind beamforming vector estimation schemes. First, note that a blind estimate of the receive beamforming vector can be computed as the dominant eigenvector of the (estimated) received covariance matrix $R_y \triangleq \sum_{i=1}^{N} \mathbf{y}_i\mathbf{y}_i^H$, i.e., as the solution to

$$\max_{\mathrm{sub.to}\|\mathbf{u}\|^2=1} \mathbf{u}^H R_y \mathbf{u}. \tag{3}$$

The covariance matrix R_y can be estimated from the blind symbols (without the need for any pilots) and thus represents the statistical information available from the received data and pilot symbols. To robustify the training estimate, therefore, we optimize the above cost function by searching for $\mathbf{u}$ in a *confidence sphere* of radius ϵ_r centered at $\hat{\mathbf{u}}_c$. This ensures that the final estimate will not deviate significantly from $\hat{\mathbf{u}}_c$, while leveraging on the information obtained from the blind statistics. Thus, with proper choice of ϵ_r, it can yield a better estimate than the pilots-only based conventional scheme, as will be shown through simulations. The new optimization problem is now stated as,

$$\max_{\substack{\mathrm{sub.to}\|\mathbf{u}\|^2=1,\\ \|\mathbf{u}-\hat{\mathbf{u}}_c\|^2\leq\epsilon_r}} \mathbf{u}^H R_y \mathbf{u}. \tag{4}$$

The above optimization problem can be solved by employing a procedure along the lines of [8], and is described below. Define $\delta_r \triangleq 2 - \epsilon_r$. Then, the confidence-sphere constraint implies

$$\delta_r \leq \mathbf{u}^H\hat{\mathbf{u}}_c + \hat{\mathbf{u}}_c^H\mathbf{u}. \tag{5}$$

Thus $\hat{\mathbf{u}}$, the estimate of the receive beamforming vector $\mathbf{u}_1$ is given as $\mathbf{u} = \arg\max g_1(\mathbf{u}, \lambda, \mu)$, where the Lagrangian cost function $g_1(\mathbf{u}, \lambda, \mu)$ is given as,

$$\mathbf{u}^H R_y\mathbf{u} + \lambda\left(\delta_r - \left(\mathbf{u}^H\hat{\mathbf{u}}_c + \hat{\mathbf{u}}_c^H\mathbf{u}\right)\right) + \mu\left(1 - \mathbf{u}^H\mathbf{u}\right).$$

Computing the complex derivative [9] with respect to $\mathbf{u}^H$, we have $(R_y - \mu I)\,\mathbf{u} - \lambda \hat{\mathbf{u}}_c = 0$, which can be simplified as,

$$\mathbf{u_r} = (R_y - \mu I)^{-1} \lambda \hat{\mathbf{u}}_c. \qquad (6)$$

Substituting in $g_1(\mathbf{u}, \lambda, \mu)$, the dual problem reduces to the objective function given as $g_2(\lambda, \mu)$, which simplifies to,

$$g_2(\lambda, \mu) = -\lambda^2 \hat{\mathbf{u}}_c^H (R_y - \mu I)^{-1} \hat{\mathbf{u}}_c + \lambda \delta_r + \mu.$$

Differentiating with respect to λ and equating to 0, the Lagrange multiplier λ is given as,

$$\lambda = \frac{\delta_r}{2\hat{\mathbf{u}}_c^H (R_y - \mu I)^{-1} \hat{\mathbf{u}}_c}.$$

Substituting in $g_2(\lambda, \mu)$, we derive the dual function for μ-optimization, $g_3(\mu)$ as,

$$g_3(\mu) = \frac{\delta_r^2}{4\hat{\mathbf{u}}_c^H (R_y - \mu I)^{-1} \hat{\mathbf{u}}_c} + \mu.$$

Differentiating with respect to μ and setting to zero,

$$\frac{\partial g_3(\mu)}{\partial \mu} = 1 - \frac{\delta_r^2 \hat{\mathbf{u}}_c^H (R_y - \mu I)^{-2} \hat{\mathbf{u}}_c}{4\left(\hat{\mathbf{u}}_c^H (R_y - \mu I)^{-1} \hat{\mathbf{u}}_c\right)^2} = 0, \qquad (7)$$

which after further simplification reduces to,

$$\mu \hat{\mathbf{u}}_c^H (R_y - \mu I)^{-2} \hat{\mathbf{u}}_c - \frac{4}{\delta_r^2}\left(\hat{\mathbf{u}}_c^H (R_y - \mu I)^{-1} \hat{\mathbf{u}}_c\right)^2 = 0.$$

Let the SVD of R_y be given by $R_y = U_R \Sigma_R U_R^H$, where Σ_R is a diagonal matrix with $\sigma_{R,i}$ as the i-th diagonal value. Defining $\tilde{\mathbf{u}}_c \triangleq U_R^H \hat{\mathbf{u}}_c$, we have,

$$\left(\sum_{i=1}^{r} \frac{|\tilde{\mathbf{u}}_c(i)|^2}{(\sigma_{R,i} - \mu)^2}\right) - \frac{4}{\delta_r^2}\left(\sum_{j=1}^{r} \frac{|\tilde{\mathbf{u}}_c(j)|^2}{\sigma_{R,i} - \mu}\right)^2 = 0. \qquad (8)$$

The above equation has to be numerically solved to find the value of μ. Finally, $\hat{\mathbf{u}}$ is obtained by substituting λ, μ in (6). Thus, one can compute the robust semi-blind estimate of the receive beamforming vector $\hat{\mathbf{u}}_c$.

IV. Transmit Beamforming Vector Estimation

We now address the issue of estimating the transmit beamforming vector $\mathbf{v}_1$. Initially, assume that the receive beamforming vector $\mathbf{u}_1$ and σ_1 (from the SVD of the channel) are known. Later, we will replace $\mathbf{u}_1$ by its robust estimate. We will also be able to show that the robust estimate of $\mathbf{u}_1$ does not depend on the value of σ_1. The received training data is given by $Y_p = HX_p + N_p$. Let us define

$$\tilde{\mathbf{y}}_p \triangleq \frac{X_p Y_p^H \mathbf{u}_1}{\gamma_p \sigma_1}. \qquad (9)$$

Note that, in the absence of noise, $\tilde{\mathbf{y}}_p = \mathbf{v}_1$. Also, if we replace $\mathbf{u}_1$ by $\hat{\mathbf{u}}_c$ above, we get $\tilde{\mathbf{y}}_p = \hat{\mathbf{v}}_c$. We seek to find the estimate of $\mathbf{v}_1$ as the solution to the least squares cost function given by the expression,

$$\hat{\mathbf{v}}_s = \arg \min_{\mathbf{v} \in \mathbb{C}^t,\ \|\mathbf{v}\|=1} \|\tilde{\mathbf{y}}_p - \mathbf{v}\|^2, \qquad (10)$$

where $\hat{\mathbf{v}}_s$ denotes the semi-blind estimate of $\mathbf{v}_1$. If X_p satisfies $X_p X_p^H = \gamma_p I_t$, the least squares estimate of $\mathbf{v}_1$ (under $\|\mathbf{v}_1\| = 1$) given perfect knowledge of $\mathbf{u}_1$ is [7],

$$\hat{\mathbf{v}}_s = \frac{X_p Y_p^H \mathbf{u}_1}{\|X_p Y_p^H \mathbf{u}_1\|}. \qquad (11)$$

Note that the above equation was derived assuming perfect knowledge of $\mathbf{u}_1$ at the receiver. In practice, there are two ways to compute the robust estimate of the transmit beamforming vector. First, we could substitute the robust estimate of $\mathbf{u}_1$ from (6) to obtain a robust estimate of $\mathbf{v}$ as

$$\hat{\mathbf{v}}_{\mathbf{r},1} = \frac{X_p Y_p^H \mathbf{u_r}}{\|X_p Y_p^H \mathbf{u_r}\|}. \qquad (12)$$

Alternatively, one can use the estimate robustification approach followed in the previous section to derive the robust semi-blind estimate of the transmit beamforming vector. For this, we restrict the estimate $\hat{\mathbf{v}}_{\mathbf{r},2}$ to lie within a confidence sphere of radius ϵ_t around $\hat{\mathbf{v}}_c$. The Lagrangian optimization problem we want to solve can now be stated as,

$$\hat{\mathbf{v}}_{\mathbf{r},2} = \arg \min_{\substack{\mathbf{v} \in \mathbb{C}^t,\ \|\mathbf{v}\|=1 \\ \|\mathbf{v} - \hat{\mathbf{v}}_c\|^2 \le \epsilon_t}} \|\tilde{\mathbf{y}}_p - \mathbf{v}\|^2, \qquad (13)$$

where $\tilde{\mathbf{y}}_p$ is obtained by replacing $\mathbf{u}_1$ by its robust estimate $\mathbf{u_r}$ in (9). To solve this problem, we start with the Lagrangian objective function $\min_{\mathbf{v},\lambda,\mu} g_1(\mathbf{v}, \lambda, \mu)$ defined as,

$$\|\tilde{\mathbf{y}}_p - \mathbf{v}\|^2 + \lambda\left(\delta_t - \left(\mathbf{v}^H \hat{\mathbf{v}}_c + \hat{\mathbf{v}}_c^H \mathbf{v}\right)\right) + \mu\left(1 - \mathbf{v}^H \mathbf{v}\right),$$

where $\delta_t \triangleq 2 - \epsilon_t$. It is shown in Appendix A that the robust estimate of $\mathbf{v}_1$ is given by

$$\hat{\mathbf{v}}_{\mathbf{r},2} = \frac{\tilde{\mathbf{y}}_p + \lambda \hat{\mathbf{v}}_c}{1 - \mu}, \qquad (14)$$

where, the Lagrangian multipliers λ and μ are computed as follows. The quantity λ is given as,

$$\lambda = \frac{1}{2}\left(\delta_t (1 - \mu) - \alpha\right), \qquad (15)$$

where $\alpha \triangleq \hat{\mathbf{v}}_c^H \tilde{\mathbf{y}}_p + \tilde{\mathbf{y}}_p^H \hat{\mathbf{v}}_c$. Next, μ is given by

$$\mu = 1 - \sqrt{\frac{\tilde{\mathbf{y}}_p^H \tilde{\mathbf{y}}_p - \frac{1}{4}\alpha^2}{1 - \frac{1}{4}\delta_t^2}}. \qquad (16)$$

It is interesting to observe that the expression for $\hat{\mathbf{v}}_{\mathbf{r},2}$ in (14) is similar in form to the *linear combination semi-blind* (LCSB) estimator presented in [7]. Thus, the robust estimator is indeed a linear combiner of the training-only estimate $\hat{\mathbf{v}}_c$ and the semi-blind estimate $\tilde{\mathbf{y}}_p$, analytically justifying the heuristic estimator proposed in [7]. This expression can be re-written in a more insightful form by substituting (16) and (15) in (14) as,

$$\hat{\mathbf{v}}_{\mathbf{r},2} = \left(\sqrt{1 - \frac{\delta_t^2}{4}}\right) \frac{\tilde{\mathbf{y}}_p - \frac{\alpha}{2}\hat{\mathbf{v}}_c}{\|\tilde{\mathbf{y}}_p - \frac{\alpha}{2}\hat{\mathbf{v}}_c\|} + \frac{\delta_t}{2}\hat{\mathbf{v}}_c, \qquad (17)$$

which is a weigted sum of two *orthogonal* vectors $\hat{\mathbf{v}}_c$ and $\tilde{\mathbf{y}}_p - \frac{\alpha}{2}\hat{\mathbf{v}}_c$. Thus, the robust estimator can intuitively be

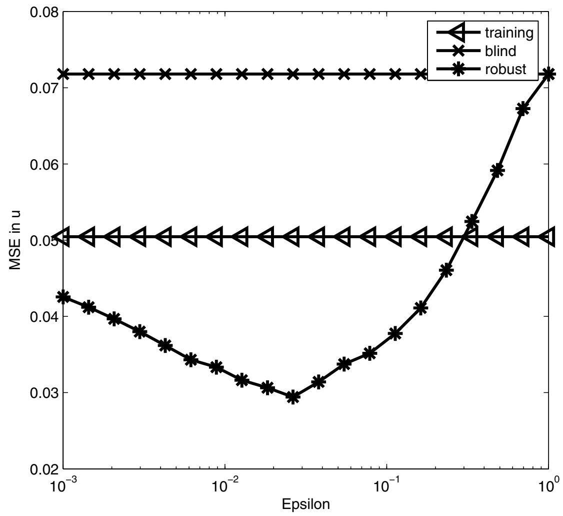

Fig. 1. MSE in $\mathbf{u}$ vs ϵ_r, 4×4 system.

thought of as correcting for the error in the training estimate $\hat{\mathbf{v}}_c$ by scaling and the addition of an orthogonal vector.

It is clear that the estimate $\hat{\mathbf{v}}_{\mathbf{r},1}$ is independent of σ_1. It can be demonstrated that the estimate $\hat{\mathbf{v}}_{\mathbf{r},2}$ is independent of σ_1, thus, we do not need to know (or estimate) σ_1 to compute it. It is also possible to show that the two methods to estimate the transmit beamforming vector given by (12) and (14) perform almost exactly the same. This is intuitively satisfying, since the robust estimate of $\mathbf{u}_1$ in (9) can be expected to yield a robust estimate of $\mathbf{v}_1$ that lies inside an ϵ confidence sphere centered at $\hat{\mathbf{v}}_c$, even though $\hat{\mathbf{v}}_{\mathbf{r},1}$ did not explicitly impose that constraint. Finally, note that the lower the estimation error in the training-based estimate, the lower is the estimation error in the robust estimate.

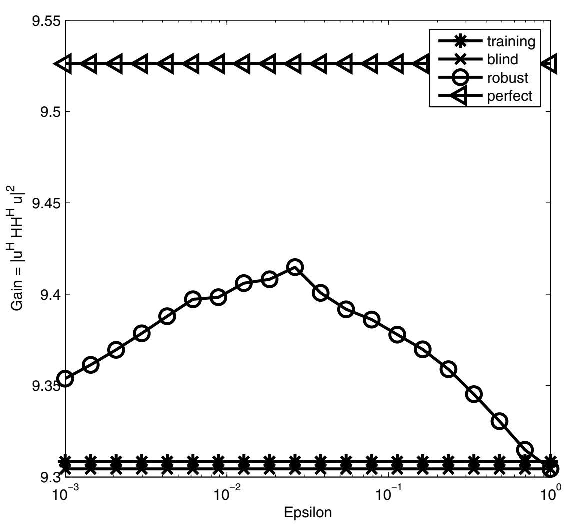

Fig. 2. Gain with $\mathbf{u}$ vs ϵ_r.

V. Simulation Results

Our simulation setup consists of a 4×4 Rayleigh flat-fading channel. For the purposes of illustration, we choose the system parameters as $P_T = 3$dB, $L = 64, N = 64$. We use 1000 random channel instantiations for the averaging, and plot the performance both in terms of the MSE in the received beamforming vector and in terms of the channel gain obtained by employing the robust estimates.

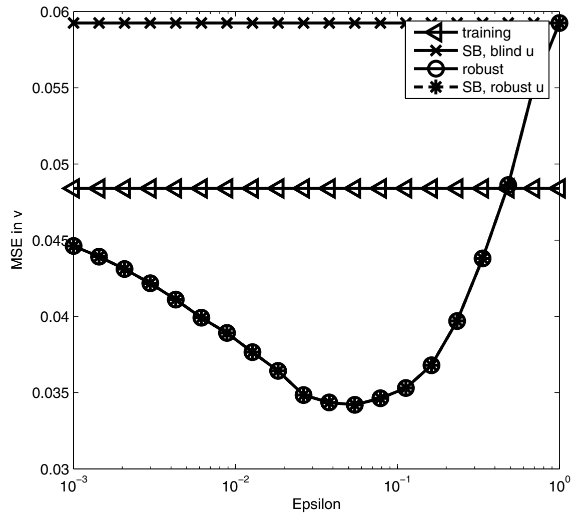

Fig. 3. MSE in $\mathbf{v}$ vs ϵ_t. The MSE from the two estimates given by (12) and (14) (labeled *robust* and *SB, robust* $\mathbf{u}$ respectively), are on top of each other.

Experiment 1: In this experiment, we illustrate the performance of the receive beamforming vector estimation algorithm described in Section III. We compare the performance of the proposed robust semi-blind algorithm with conventional (training-based) estimation, exclusively blind estimation and also with perfect (genie) estimation, in terms of the MSE in the estimate (in Fig. 1), and in terms of the channel gain (in Fig. 2), which we define as $\mathbf{u}^H HH^H \mathbf{u}$, since $\mathbf{u}$ is the dominant left singular vector of H. The MSE and the gain are plotted versus the radius of the confidence ball ϵ. It is clear that the robust estimator outperforms both the training-based estimate as well as the exclusively blind estimate for a certain range of ϵ. In general, as $\epsilon \to 0$, the performance of the robust estimator tends to that of the training-based estimate, and as $\epsilon \to 1$, the performance defaults to that of the blind estimate given by the dominant eigenvector of R_y.

Experiment 2: Next, we illustrate the performance of the transmit beamforming vector estimation problem. As before, we plot the MSE (in Fig. 3) and gain (in Fig. 4) of the channel versus the radius of the confidence ball ϵ. We compare the two robust methods given by (12) and (14), as well as a simple Semi-Blind (SB) estimator obtained from (11) by replacing $\mathbf{u}_1$ by its blind estimate (the dominant eigenvector of the covariance matrix R_y). In this experiment, however, the gain is defined as $\mathbf{v}^H H^H H\mathbf{v}$, since $\mathbf{v}$ is the dominant right singular vector of H. Note that the two methods of forming the robust estimate of $\mathbf{v}_1$ perform almost exactly the same, as expected. Also note that as $\epsilon \to 0$, the performance of the robust estimator tends to that of the training-based estimate,

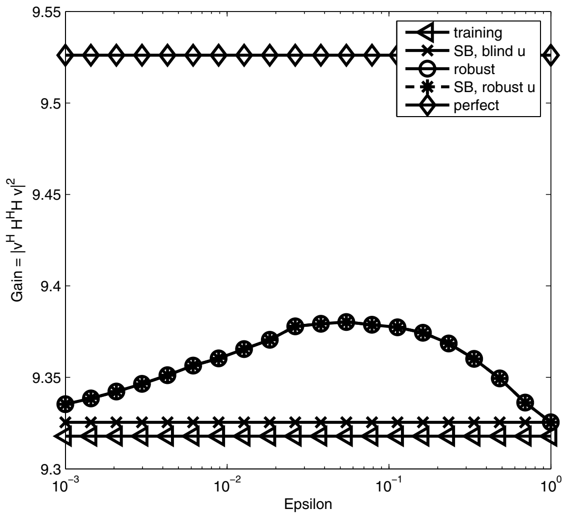

Fig. 4. Gain with $\mathbf{v}$ vs ϵ_t. The two robust estimates labeled *robust* and *SB, robust* $\mathbf{u}$ given by (12) and (14), respectively are right on top of each other.

and as $\epsilon \rightarrow 1$, the performance of the robust estimator defaults to that of the semi-blind estimation scheme given by (11) with $\mathbf{u}_1$ replaced by its blind estimate, again as expected.

VI. Conclusions

We have investigated robust semi-blind beamforming vector estimation for MRT based MIMO communication. We have derived robust semi-blind estimates for both the receive and transmit beamforming vectors, where the robustness is guaranteed by requiring that the estimated beamforming vector lie within a confidence sphere centered at the preliminary training based estimate. The robust estimation scheme is shown to perform as good or better than the training only or previous semi-blind schemes, thus making it a promising technique for wireless channel estimation. How to choose the radius of the confidence sphere is an important topic for future research.

VII. Appendix

A. Derivation of (14)

Recall that we want to minimize $g_1(\mathbf{v}, \lambda, \mu)$ given by

$$\|\tilde{\mathbf{y}}_p - \mathbf{v}\|^2 + \lambda\left(\delta_t - \left(\mathbf{v}^H \hat{\mathbf{v}}_c + \hat{\mathbf{v}}_c^H \mathbf{v}\right)\right) + \mu\left(1 - \mathbf{v}^H \mathbf{v}\right),$$

where $\delta_t \triangleq 2 - \epsilon_t$, with respect to $\mathbf{v}$, λ and μ. Taking the complex partial derivative with respect to $\mathbf{v}^H$ and setting to zero, we have

$$\left.\frac{\partial g_1(\mathbf{v}, \lambda, \mu)}{\partial \mathbf{v}^H}\right|_{\mathbf{v}=\mathbf{v}_r} = -\left(\tilde{\mathbf{y}}_p - \mathbf{v}_r\right) - \lambda \hat{\mathbf{v}}_c - \mu \mathbf{v}_r = 0,$$

whence

$$\mathbf{v}_r = \frac{\tilde{\mathbf{y}}_p + \lambda \hat{\mathbf{v}}_c}{1-\mu}.$$

Substituting in $g_1(\mathbf{v}, \lambda, \mu)$ and simplyfying, we get $g_2(\lambda, \mu)$, the objective of the dual maximization problem:

$$g_2(\lambda, \mu) \triangleq \tilde{\mathbf{y}}_p^H \tilde{\mathbf{y}}_p - \frac{\left(\tilde{\mathbf{y}}_p + \lambda \hat{\mathbf{v}}_c\right)^H \left(\tilde{\mathbf{y}}_p + \lambda \hat{\mathbf{v}}_c\right)}{1-\mu} + \mu + \lambda \delta_t. \quad (18)$$

Differentiating with respect to λ and setting to zero, we have

$$\frac{\partial g_2(\lambda, \mu)}{\partial \lambda} = \delta_t - \frac{\hat{\mathbf{v}}_c^H \tilde{\mathbf{y}}_p + \tilde{\mathbf{y}}_p^H \hat{\mathbf{v}}_c + 2\lambda \hat{\mathbf{v}}_c^H \hat{\mathbf{v}}_c}{1-\mu} = 0, \quad (19)$$

which yields

$$\lambda = \frac{1}{2}\left(\delta_t(1-\mu) - \hat{\mathbf{v}}_c^H \tilde{\mathbf{y}}_p - \tilde{\mathbf{y}}_p^H \hat{\mathbf{v}}_c\right). \quad (20)$$

Thus, we have the final maximization over μ as follows. For notational simplicity, define $\alpha \triangleq \hat{\mathbf{v}}_c^H \tilde{\mathbf{y}}_p + \tilde{\mathbf{y}}_p^H \hat{\mathbf{v}}_c$ and $\Gamma = 1 - \mu$. Then, the maximization of the dual of $g_2(\lambda, \mu)$ can be rewritten in terms of maximization of $g_3(\Gamma)$ given by

$$\frac{\delta_t}{2}\left(\delta_t \Gamma - \alpha\right) + (1-\Gamma) - \frac{\left[\tilde{\mathbf{y}}_p^H \tilde{\mathbf{y}}_p + \frac{1}{4}\left(\delta_t^2 \Gamma^2 - \alpha^2\right)\right]}{\Gamma}. \quad (21)$$

Differentiating with respect to Γ and setting to zero, we get

$$\frac{\delta_t^2}{2} - 1 + \frac{1}{\Gamma^2}\left[\tilde{\mathbf{y}}_p^H \tilde{\mathbf{y}}_p + \frac{1}{4}\left(\delta_t^2 \Gamma^2 - \alpha^2\right)\right] - \frac{1}{\Gamma}\left[\frac{1}{4} 2 \delta_t^2 \Gamma\right] = 0,$$

which yields

$$\Gamma^2 = \frac{\tilde{\mathbf{y}}_p^H \tilde{\mathbf{y}}_p - \frac{1}{4}\alpha^2}{1 - \frac{1}{4}\delta_t^2}. \quad (22)$$

It can be seen that the second derivative is

$$\frac{\partial^2 g_3(\Gamma)}{\partial \Gamma^2} = \frac{-2\left(\tilde{\mathbf{y}}_p^H \tilde{\mathbf{y}}_p - \frac{\alpha^2}{4}\right)}{\Gamma^3} \quad (23)$$

Thus, when the positive square root is chosen for Γ, $\frac{\partial^2 g_3(\Gamma)}{\partial \Gamma^2} < 0$ and g_3 is guaranteed to be at a local maximum. From this, we can determine μ as

$$\mu = 1 - \sqrt{\Gamma^2}. \quad (24)$$

Substituting for λ and μ from (20) and (24) into (18) and simplifying, we obtain the expression for the robust estimate in (14) for the transmit beamforming vector.

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