# Semi-Definite Programming (SDP) Relaxation Based Semi-Blind Channel Estimation for Frequency-Selective MIMO MC-CDMA Systems

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***Abstract*—In this paper, we propose a semi-definite programming (SDP) relaxation based semi-blind (SRSB) channel estimation scheme for frequency-selective multiple-input multiple-output (MIMO) multi-carrier code division for multiple access (MC-CDMA) systems. The SRSB scheme is derived from the multi-path multi-carrier decorrelator (MMD) based robust channel estimation framework developed for MIMO MC-CDMA. We formulate the semi-blind channel estimation scenario for MIMO MC-CDMA systems as a convex quadratic programming (QP) problem. Subsequently with an approximate rank relaxation this is recast as an equivalent SDP problem. Hence, the SDP solvers utilized in maximum likelihood MIMO symbol detection can be utilized to obtain the semi-blind channel estimate, thereby potentially enhancing the estimation performance without the need for additional computational resources. Simulation results demonstrate that the mean-squared error (MSE) performance of the SDP relaxation based semi-blind estimation is significantly superior compared to that of the conventional training based least-squares estimator.**

## I. Introduction

The accelerated pace of demand for high-speed data access over wireless channels has garnered significant research interest in multiple-input multiple-output (MIMO) technology and spread-spectrum systems such as multi-carrier code division for multiple access (MC-CDMA) communications. MC-CDMA synergistically harnesses the advantages of CDMA, which combats wireless channel fading through multi-path diversity combining and orthogonal frequency division multiplexing (OFDM), which converts the frequency-selective wireless communication channel into multiple parallel narrowband flat-fading channels [1]. MIMO technology can be employed in MC-CDMA systems to further increase the data rate through spatial multiplexing. However, these performance gains of MIMO MC-CDMA systems are critically dependent on the accuracy of the channel estimates. This entails for channel estimation paradigms that estimate the wireless channel coefficients accurately with reasonable computational complexity. Although, traditional training based channel estimation schemes have a low computational complexity, they result in a high communication overhead [2] as training symbols do not convey information. Recently, several techniques based on the principles of robust Capon beamforming have been proposed to obtain superior channel estimates. A robust approach to channel estimation and multi-user detection (MUD) for MC-CDMA is presented in [3]. However, the work is restricted to single-input single-output (SISO) systems and cannot be readily extended to the MIMO scenario. The multi-path multi-carrier decorrelator (MMD) based framework developed for semi-blind MIMO MC-CDMA in [4] can be employed to widen the scope of robust channel estimation to frequency selective MIMO MC-CDMA systems. Several works on low complexity algorithms for MIMO maximum likelihood detection based on the semi-definite programming (SDP) relaxation have been proposed [5]–[7].

In this paper, we propose a SDP relaxation based semi-blind (SRSB) channel estimation scheme for frequency-selective MIMO MC-CDMA systems. We formulate the robust channel estimation (RCB) for MIMO as a convex quadratic programming problem using the MMD framework. We then recast it as an equivalent SDP through an appropriate relaxation. This SDP relaxation based channel estimation scheme offers a significant reduction of the computational resources required in systems which employ SDP solvers for maximum likelihood detection. Hence, the SDP solvers utilized in symbol detection can also be used to obtain a robust semi-blind channel estimate, thereby enhancing the accuracy of estimation without the need for additional computational resources. Simulation results demonstrate that the proposed SRSB estimation scheme has superior performance over the conventional schemes. The rest of the paper is organized as follows. In section II, we describe the frequency-selective MIMO MC-CDMA system model. Section III describes the proposed SRSB channel estimation scheme for MIMO MC-CDMA system. Simulation results are given in section IV and we present our conclusions with section V.

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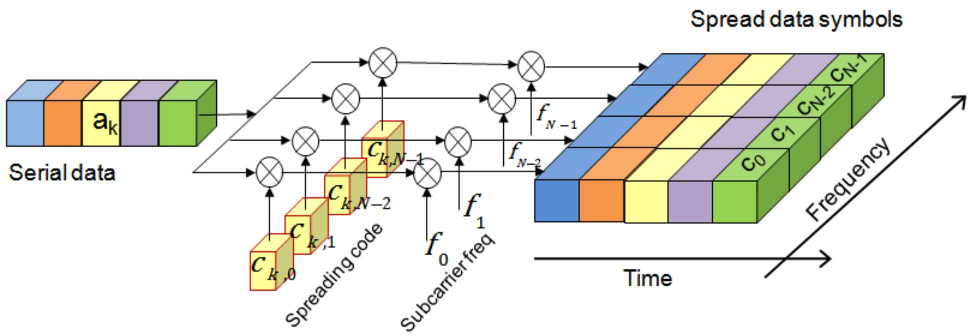


Fig. 1. Schematic of MC-CDMA wireless transmission

## II. System Model

Consider a downlink (DL) MIMO MC-CDMA system with $N_r$ receive antennas and $N_t$ transmit antennas. The base-band model of the frequency-selective MIMO MC-CDMA system is given as,

$$\mathbf{y}(n) = \sum_{i=0}^{L_h-1} \mathbf{H}(i)\mathbf{s}(n-i) + \mathbf{v}(n) \tag{1}$$

where $\mathbf{y}(n) \in \mathbb{C}^{N_r \times 1}$ is the received signal at time instant $n$, $\mathbf{s}(n) \in \mathbb{C}^{N_t \times 1}$ is the composite DL transmit vector at time instant $n$, $\mathbf{v}(n) \in \mathbb{C}^{N_r \times 1}$ is additive spatio-temporally white Gaussian noise with covariance $\mathrm{E}\left\{\mathbf{v}(n)\mathbf{v}(n)^H\right\} = \sigma_n^2 \mathbf{I}_{N_r}$. Each $\mathbf{H}(i) \in \mathbb{C}^{N_r \times N_t}$, $0 \leq i \leq L_h - 1$, is the channel matrix corresponding to the $i^{th}$ lag and $L_h$ is the length of the MIMO frequency-selective finite impulse response (FIR) channel. Each complex element $h_{r,t}(i)$ of the matrix $\mathbf{H}(i)$ denotes the channel coefficient between transmit antenna $t$ and receive antenna $r$ corresponding to the $i^{th}$ delay. Let $K$ denote the total number of DL users of the MIMO MC-CDMA system. Let the symbol vector of $k^{th}$ user, transmitted in the $p^{\text{th}}$ MC block, be denoted by $\mathbf{a}_k(p) = [a_{k,1}(p), a_{k,2}(p), \ldots, a_{k,N_t}(p)]^T$, where $a_{k,t}(p)$ is the symbol corresponding the $t^{th}$ transmit antenna. Let the covariance of the transmit symbol vector $\mathbf{a}_k(p)$ be given as $\mathrm{E}\left\{\mathbf{a}_k(p)\mathbf{a}_k^H(p)\right\} = P_d \mathbf{I}_{N_t}$ where $P_d$ is the transmit power corresponding to each transmit antenna. The DL transmit symbol vector loaded onto the $m^{th}$ subcarrier in this multi-carrier system is given by $\mathbf{X}_m(p) = \sum_{k=0}^{K-1} c_{k,m}\mathbf{a}_k(p)$, where $\mathbf{X}_m(p) \in \mathbb{C}^{N_t \times 1}$, $\{c_{k,m}\}_{m=0}^{N-1}$ is the spreading code of the $k^{th}$ user and the spreading length $N$ is equal to the number of subcarriers. The composite DL signal $\mathbf{s}(n)$ is given by the $N$-point IFFT of the spread data symbol vectors $\mathbf{X}_m(p)$ followed by the addition of the cyclic prefix. Hence, after serial-to-parallel conversion, removal of cyclic prefix, and $N$-point FFT at the receiver, $\mathbf{Y}_m(p) \in \mathbb{C}^{N_r \times 1}$, the received data at subcarrier $m$ is given as [8],

$$\mathbf{Y}_m(p) = \mathbf{Z}_m \mathbf{X}_m(p) + \mathbf{V}_m(p), \tag{2}$$

where $\mathbf{Z}_m \in \mathbb{C}^{N_r \times N_t}$, the flat-fading channel coefficient matrix corresponding to the $m^{th}$ subcarrier, is given by the $N$-point FFT of the MIMO frequency-selective channel $\mathbf{H}(i)$, $0 \leq i \leq L_h - 1$ as,

$$\mathbf{Z}_m = \sum_{i=0}^{L_h-1} \mathbf{H}(i) e^{-j2\pi i \frac{m}{N}}. \tag{3}$$

The quantity $\mathbf{V}_m(p)$ is the FFT of the receiver noise vectors $\mathbf{v}(n)$. We employ the optimal multi-path multi-carrier decorrelation (MMD) receiver presented in [4] which reduces the $L_h$ lag frequency-selective MIMO MC-CDMA channel to that of an equivalent $N_r L_h \times N_t$ flat-fading MIMO channel. Consider the signal detection at the $0^{th}$ user, with the remaining $K-1$ $(1 \leq k \leq K-1)$ users considered as interferers. The received data $Y_{r,m}(p)$ at user 0 can be decorrelated with the spreading code $\{c_{0,m}\}_{m=0}^{N-1}$ to obtain the optimal MMD statistic $d_{r,l}(p)$ corresponding to receive antenna $r$ and delay $l$, which is given as,

$$d_{r,l}(p) = \frac{1}{N} \sum_{m=0}^{N-1} Y_{r,m}(p) c_{0,m}^* e^{j2\pi l \frac{m}{N}}. \tag{4}$$

Stacking the decision statistics for the $L_h$ lags, $0 \leq l \leq L_h - 1$, the $N_r L_h$ dimensional decision statistic for the frequency-selective MIMO MC-CDMA system model is given as,

$$\mathbf{d}(p) \triangleq \left[\mathbf{d}_0^T(p), \mathbf{d}_1^T(p), \ldots, \mathbf{d}_{L_h-1}^T(p)\right]^T = \boldsymbol{\mathcal{H}}\mathbf{a}_0(p) + \widetilde{\mathbf{v}}(p), \tag{5}$$

where the block matrix $\boldsymbol{\mathcal{H}} \in \mathbb{C}^{N_r L_h \times N_t}$ is defined as

$$\boldsymbol{\mathcal{H}} \triangleq \left[\mathbf{H}^T(0), \mathbf{H}^T(1), \ldots, \mathbf{H}^T(L_h - 1)\right]^T. \tag{6}$$

The noise vector $\widetilde{\mathbf{v}}(p) \in \mathbb{C}^{N_r L_h \times 1}$ is Gaussian with covariance $\mathrm{E}\{\widetilde{\mathbf{v}}(p)\widetilde{\mathbf{v}}^H(p)\} = \frac{\sigma_n^2}{N}\mathbf{I}_{N_r L_h}$.

### A. Training based Least-Squares (LS) Channel Estimation

Let the matrix $\mathbf{A}_p \in \mathbb{C}^{N_t \times L_p}$, the pilot symbol matrix corresponding to the $L_p$ pilot transmissions $\mathbf{A}_p = [\mathbf{a}(1), \mathbf{a}(2), ..., \mathbf{a}(L_p)]$, be such that $\mathbf{A}_p\mathbf{A}_p^H = L_p P_t \mathbf{I}$, where $P_t$ is training power. The MMD output matrix $\mathbf{D}_p = [\mathbf{d}(1), \mathbf{d}(2), ..., \mathbf{d}(L_p)]$, where $\mathbf{D}_p \in \mathbb{C}^{N_r \times L_p}$, corresponding to the transmission of the $L_P$ pilot symbols is given from (5) as,

$$\mathbf{D}_p = \boldsymbol{\mathcal{H}}\mathbf{A}_p + \widetilde{\mathbf{V}}_p, \tag{7}$$

where the matrix $\widetilde{\mathbf{V}}_p \in \mathbb{C}^{N_r \times L_p}$ corresponds to the noise at the receiver. From the expression for the pilot symbol output matrix $\mathbf{D}_p$ given in (7) it can be seen that the maximum-likelihood (ML) training estimate $\widehat{\boldsymbol{\mathcal{H}}}_{\mathrm{T}}$ of the frequency-selective MIMO MC-CDMA channel matrix $\boldsymbol{\mathcal{H}}$ is given by the standard LS estimator [9],

$$\widehat{\boldsymbol{\mathcal{H}}}_{\mathrm{T}} = \mathbf{D}_p \mathbf{A}_p^{\dagger} = \boldsymbol{\mathcal{H}} + \widetilde{\mathbf{V}}\mathbf{A}_p^{\dagger} = \boldsymbol{\mathcal{H}} + \frac{1}{L_p P_t}\widetilde{\mathbf{V}}\mathbf{A}_p^{H}, \quad (8)$$

where the last equality follows from the fact that $\mathbf{A}_p^{\dagger}$, the pseudo-inverse of pilot symbol matrix $\mathbf{A}_p$ is given as $\mathbf{A}_p^{\dagger} = \mathbf{A}_p^H \left(\mathbf{A}_p \mathbf{A}_p^H\right)^{-1}$ and $\mathbf{A}_p \mathbf{A}_p^H = L_p P_t \mathbf{I}$. Hence, the CRB on the MSE, which is achieved by the optimal LS estimator above, is given by,

$$\mathrm{MSE}_T = \mathrm{E}\{\|\widehat{\boldsymbol{\mathcal{H}}}_{\mathrm{T}} - \boldsymbol{\mathcal{H}}\|_F^2\} = \frac{N_t N_r L_h \sigma_{\tilde{v}}^2}{L_p P_t}. \quad (9)$$

Next, we derive the SDP relaxation based semi-blind estimation scheme for frequency-selective MIMO MC-CDMA systems.

## III. Semi-Definite Relaxation Based Semi-Blind Channel Estimation Scheme

The robust Capon beamforming scheme proposed in [10] considers the array steering vector to lie in an uncertainty set that is modeled as an ellipsoid. The array output signal power is maximized such that the beamformer belongs to an appropriate ellipsoidal uncertainty set. From the discussion in [10], $\widehat{\boldsymbol{\mathcal{H}}}_R$, the uncertainty set based robust estimate of the channel matrix $\boldsymbol{\mathcal{H}}$ at the output of the MMD MIMO MC-CDMA receiver, is given as the solution of the optimization problem,

$$\begin{aligned} \min_{\boldsymbol{\mathcal{H}}_R}.\quad & \mathrm{tr}\left(\boldsymbol{\mathcal{H}}_R^H \mathbf{R}_d^{-1} \boldsymbol{\mathcal{H}}_R\right) \\ \text{s.t.}\quad & \|\widehat{\boldsymbol{\mathcal{H}}}_T - \boldsymbol{\mathcal{H}}_R\|_F^2 \le \epsilon, \end{aligned} \quad (10)$$

where the $\mathbf{R}_d \in \mathbb{C}^{N_r L_h \times N_r L_h}$ is the covariance matrix of the MMD output vectors $\mathbf{d}(p)$ and $\mathrm{tr}(\cdot)$ is the trace of a matrix. Further, the covariance matrix $\mathbf{R}_d$ is estimated from the blind output symbols as,

$$\widehat{\mathbf{R}}_d = \frac{1}{N_b}\sum_{p=1}^{N_b} \mathbf{d}^b(p)(\mathbf{d}^b(p))^H, \quad (11)$$

and $\widehat{\boldsymbol{\mathcal{H}}}_T$ is the least squares estimate of $\boldsymbol{\mathcal{H}}$, as derived in section II-A. Hence, in principle, this scheme is semi-blind in nature as it employs the initial channel estimate obtained through training based estimation and the statistical information of the received data. It can be seen that the above objective function $\mathrm{tr}\left(\boldsymbol{\mathcal{H}}_R^H \mathbf{R}_d^{-1} \boldsymbol{\mathcal{H}}_R\right)$ can be analogously expressed as,

$$\mathrm{tr}(\mathbf{CX}) = \mathrm{tr}(\mathbf{R}_d^{-1}\boldsymbol{\mathcal{H}}_R \boldsymbol{\mathcal{H}}_R^H + \mathbf{I}_{N_t}) = \mathrm{tr}(\boldsymbol{\mathcal{H}}_R^H \mathbf{R}_d^{-1}\boldsymbol{\mathcal{H}}_R + \mathbf{I}_{N_t}),$$

where the matrices $\mathbf{C} \in \mathbb{C}^{(N_r L_h + N_t)\times(N_r L_h + N_t)}$ and $\mathbf{X} \in \mathbb{C}^{(N_r L_h + N_t)\times(N_r L_h + N_t)}$ are given as,

$$\mathbf{C} = \begin{bmatrix} \mathbf{R}_d^{-1} & \mathbf{0}_{N_r L_h \times N_t} \\ \mathbf{0}_{N_t \times N_r L_h} & \mathbf{I}_{N_t} \end{bmatrix}, \mathbf{X} = \begin{bmatrix} \boldsymbol{\mathcal{H}}_R \boldsymbol{\mathcal{H}}_R^H & \boldsymbol{\mathcal{H}}_R \\ \boldsymbol{\mathcal{H}}_R^H & \mathbf{I}_{N_t} \end{bmatrix}. \quad (12)$$

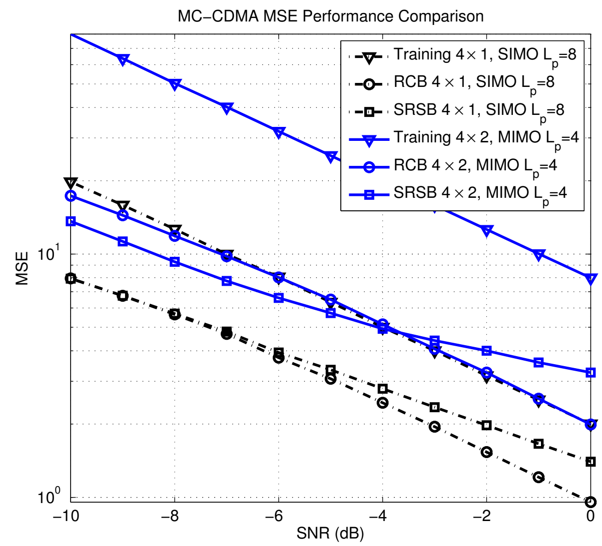


Fig. 2. MSE comparison of SRSB, robust and training based SIMO and MIMO MC-CDMA channel estimators.

Further, the constraint can be readily expressed as,

$$\mathrm{tr}\left(\boldsymbol{\mathcal{H}}_R \boldsymbol{\mathcal{H}}_R^H - \widehat{\boldsymbol{\mathcal{H}}}_T \boldsymbol{\mathcal{H}}_R^H - \boldsymbol{\mathcal{H}}_R \widehat{\boldsymbol{\mathcal{H}}}_T^H + \widehat{\boldsymbol{\mathcal{H}}}_T \widehat{\boldsymbol{\mathcal{H}}}_T^H\right) \le \epsilon.$$

Rearranging the terms above, it can be readily seen that the constraint takes the form $\mathrm{tr}(\mathbf{FX}) \le \delta$, where $\mathbf{F} \in \mathbb{C}^{(N_r L_h + N_t)\times(N_r L_h + N_t)}$ and $\delta$ are given as,

$$\mathbf{F} = \begin{bmatrix} \mathbf{I}_{N_r L_h} & -\widehat{\boldsymbol{\mathcal{H}}}_T \\ -\widehat{\boldsymbol{\mathcal{H}}}_T^H & \mathbf{I}_{N_t} \end{bmatrix}, \quad \delta \triangleq \epsilon - \mathrm{tr}(\widehat{\boldsymbol{\mathcal{H}}}_T \widehat{\boldsymbol{\mathcal{H}}}_T^H) + N_t. \quad (13)$$

Therefore, the QP problem in (10) can be equivalently recast [11] in SDP form as,

$$\begin{aligned} \min_{\mathbf{X}}.\quad & \mathrm{tr}(\mathbf{CX}) \\ \text{s.t.}\quad & \mathrm{tr}(\mathbf{FX}) \le \delta \\ & \mathbf{X} \succcurlyeq 0, \end{aligned} \quad (14)$$

where $\mathbf{X} \succcurlyeq 0$ indicates that $\mathbf{X}$ is constrained to lie in the convex set of positive semi-definite matrices. The solution, $\mathbf{X}^*$, of the optimization problem in (14), which is a block matrix as given in (12), is utilized to extract the SRSB channel estimate, $\widehat{\boldsymbol{\mathcal{H}}}_S$. The proposed approach can be summarized as follows

**SRSB Scheme:**

Step 1) Estimate the block channel matrix $\boldsymbol{\mathcal{H}}$ using training symbols (8) and compute the matrix $\mathbf{F}$ as shown in (13).

Step 2) Compute the covariance matrix $\mathbf{R}_d$ using the blind information symbols (11) and compute the matrix $\mathbf{C}$ as shown in (12).

Step 3) Compute the radius of the uncertainty set, $\delta$, from (9) and (13).

Step 4) Solve the SDP problem (14) with matrices $\mathbf{F}$ and $\mathbf{C}$ and radius of the uncertainty set, $\delta$, obtained in Step 1 - Step 3.

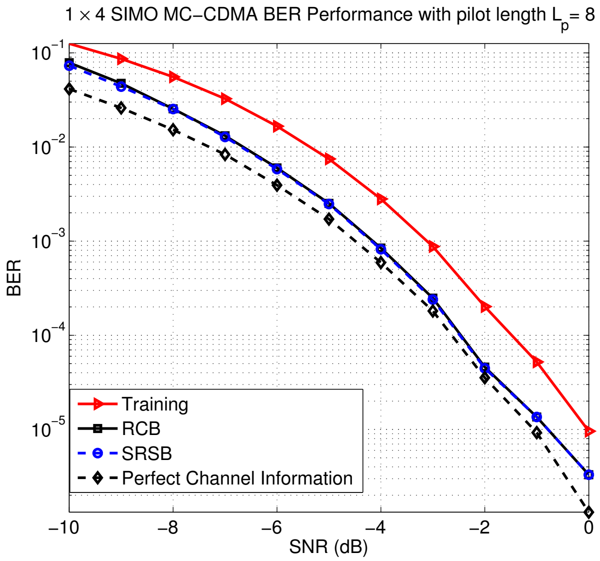


Fig. 3. BER performance comparison of the SRSB, robust and training based SIMO MC-CDMA channel estimation schemes.

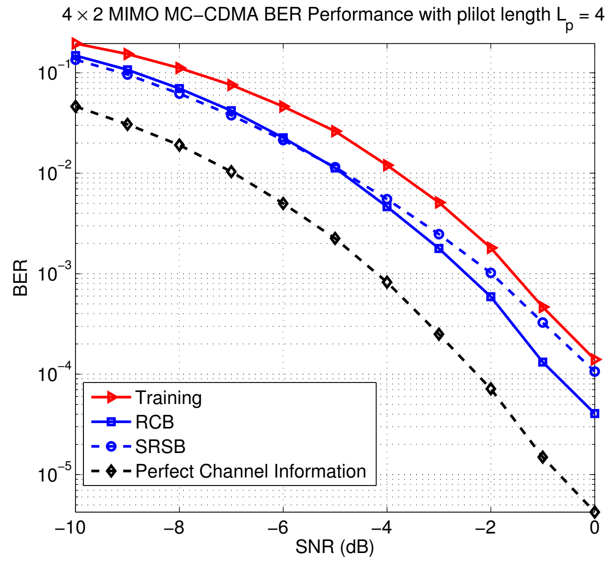


Fig. 4. BER performance comparison of the SRSB, robust and training based MIMO MC-CDMA channel estimation schemes.

Step 5) SRSB channel estimate, $\widehat{\mathcal{H}}_S$, is obtained by decomposing (12), $\mathbf{X}^*$, the solution of the SDP problem.

### A. Uncertainty based Robust Channel Estimation

The robust channel estimation scheme for a SISO MC-CDMA system based on the spherical uncertainty set is given in [3]. However, the framework presented therein is restrictive and cannot be extended to the frequency-selective MIMO MC-CDMA scenario under consideration. Employing the MMD framework [4], the RCB scheme for MIMO MC-CDMA can formulated as given in (10). The expression for the radius of the uncertainty set $\epsilon$ for the robust estimator (10) is $\epsilon = \frac{N_t N_r L_h \sigma_{\tilde{v}}^2}{L_p P_t}$. It should be observed that $\epsilon$ is related to MSE of the LS estimator (9). The optimal robust estimate $\widehat{\mathcal{H}}_R$ can be obtained through the standard KKT framework for convex optimization problems [11]. Let $f(\lambda)$ be the Lagrangian associated with (10) and $\lambda$ be the Lagrange multiplier. The Lagrangian is given as,

$$f(\lambda) = \mathrm{tr}(\mathcal{H}_R^H \mathbf{R}_d^{-1} \mathcal{H}_R) + \lambda \left( ||\widehat{\mathcal{H}}_T - \mathcal{H}_R||_F^2 - \epsilon \right). \quad (15)$$

Differentiating $f(\lambda)$ with respect to $\mathcal{H}_R$ and equating to zero,

$$\frac{\partial f(\lambda)}{\partial \mathcal{H}_R} = 2\mathbf{R}_d^{-1}\mathcal{H}_{opt} + \lambda(-2\widehat{\mathcal{H}}_T + 2\mathcal{H}_{opt}) = 0, \quad (16)$$

the optimal robust estimate is obtained as,

$$\mathcal{H}_{opt} = \left( \mathbf{I} + \frac{\mathbf{R}_d^{-1}}{\lambda} \right)^{-1} \widehat{\mathcal{H}}_T. \quad (17)$$

Employing the Woodbury matrix identity [12], the expression (17) can be expressed as,

$$\widehat{\mathcal{H}}_R = \left( \mathbf{I} - (\mathbf{I} + \lambda \mathbf{R}_d)^{-1} \right) \widehat{\mathcal{H}}_T, \quad (18)$$

where $\lambda$ is derived as the solution of $\| \left(\mathbf{I} + \lambda \mathbf{R}_d\right)^{-1} \widehat{\mathcal{H}}_T \|_F^2 = \epsilon$, similar to the estimate of the robust Capon beamformer [10]. Simulation results are presented in the next section to compare the performance of the proposed channel estimation schemes.

## IV. Simulation Results

We simulated frequency-selective $4 \times 1$ SIMO and $4 \times 2$ MIMO MC-CDMA downlink scenarios, i.e. with $N_r = 4$ receive antennas and $N_t = 1, 2$ transmit antennas respectively, with a delay spread of $L_h = 4$ . The system comprised of $K = 12$ active users with spreading sequences of length $N = 256$. The covariance matrix $\mathbf{R}_d$ is estimated from $N_b = 1000$ blind data symbols. We compare the performance of the proposed SDP based semi-blind estimation scheme (SRSB) and the robust channel estimation scheme (RCB) with that of the conventional training based estimator with pilot length $L_p = 4$ and $L_p = 8$ for the MIMO and SIMO channels respectively. In Fig.2, the mean-squared error (MSE) of the SRSB, RCB, and LS estimators is plotted against the signal-to-noise power ratio (SNR). It can be seen that the MSE performance of the SRSB scheme and the RCB scheme is significantly superior compared to that of the training based LS estimator. The MSE of the robust semi-blind MIMO MC-CDMA channel estimation schemes is lower by approximately 6 dB for the MIMO and 3 dB for the SIMO system relative to the LS estimator. However in case of the MIMO system, the SRSB scheme has a slight performance loss in comparison to the RCB scheme since the QP problem in (10) is approximated by SDP (14) by relaxing the rank constraint. In Fig.3 and Fig.4, we consider the probability of error (BER) performance for the channel estimates obtained through the competing schemes that are employed for the detection of the transmit symbol vectors. In the SIMO system, with a nominal increase in pilot symbols, the BER is close to that of perfect knowledge of channel impulse response. Hence, the SRSB estimation scheme and the robust estimator yield superior channel estimates compared to the exclusive training based LS estimator.

## V. Conclusion

A novel semi-definite relaxation based robust estimation scheme has been proposed for frequency-selective MIMO MC-CDMA channel estimation. The SDP relaxation semi-blind channel estimation yields superior channel estimates without the need for additional computational resources in systems which employ SDP solvers for maximum likelihood decoding of the transmit symbol vectors. The proposed techniques have significantly superior performance in comparison to the conventional training based channel estimation scheme.

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