SUPERIMPOSED PILOTS VS. CONVENTIONAL PILOTS FOR CHANNEL ESTIMATION

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ABSTRACT

We study two competing pilot symbol based schemes, viz. superimposed pilots (SP) and conventional pilots (CP) for the estimation of a single-input multiple-output (SIMO) wireless channel. We derive expressions for the mean-squared error (MSE) and Cramer-Rao bound (CRB) of estimation of the SIMO channel. It is demonstrated that the asymptotic CRB of SP is 3dB lower than the MSE of the simplistic mean estimator. We propose a semi-blind SP scheme which asymptotically achieves this CRB. In the second part, we quantify the throughput performance of the estimation schemes by developing a framework for the worst case capacity of a channel with correlated symbols and noise. It is observed that while CP outperforms SP in terms of MSE of estimation, SP has an overall advantage over CP in terms of net throughput and is therefore bandwidth efficient. Further, we derive expressions for the optimal source to noise power ratio (SNR) as a function of the pilot to noise power ratio (PNR) in the context of SP.

1. INTRODUCTION

Several advances have been made towards enhanced signal processing for SIMO (Single-Input Multiple-Output) and MIMO (Multiple-Input Multiple-Output) wireless systems. Availability of accurate channel knowledge in such systems can result in significant performance improvements. Traditionally, the channel has been estimated by the transmission of a known sequence of pilot symbols prior to transmitting information bearing data symbols in each estimation period. This scheme is termed as conventional pilot (CP). A major concern in this endeavor is the potential wastage of bandwidth due to the exclusive transmission of pilot symbols which bear no information. Recent advances in signal processing have suggested an innovative scheme for channel estimation using superimposed pilot (SP) symbols. SP based schemes trade off bandwidth for power and the additional power is used to transmit a repetitive sequence of pilot symbols superimposed over the data symbols. Hence, SP schemes do not sacrifice bandwidth by exclusively transmitting pilots. Schemes for SP based estimation have been explored in [1, 2, 3].

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In this work, we derive expressions for the mean-square error (MSE) of estimation of the SP and CP schemes. Further, employing Gaussian transmitted symbols, we derive the true Cramer-Rao Bound (CRB) for SP based estimation, where only approximate bounds exist in literature[1]. The simplistic first-order statistic (mean) based estimation scheme proposed in works such as [1, 2] ignores the information present in the second-order statistics. We propose a semi-blind SP estimation scheme which employs this second-order statistical information to enhance estimation accuracy. This estimate is demonstrated to have an asymptotic MSE that is 3dB lower than the mean based estimate. Another aspect of this work is the framework for throughput performance analysis. A similar study has been presented in [4]. In our work we derive a general expression for the capacity lower bound of correlated channels to analyze the throughput performance of SP and CP systems with estimation error. It is seen from this study that even though CP outperforms SP with respect to MSE of estimation, SP can outperform CP in terms of overall system throughput and hence is more suited for communication purposes. Further, we addresses the issue of optimal source power control in a system employing SP. We present closed form expressions for the optimum SNR to maximize post-processing SNR (PSNR) for different receive beamformers. Due to space limitations, some proofs will be omitted and they can be found in [5].

2. CP AND SP ESTIMATION: STATIC CHANNEL

Consider a single-input multiple-output (SIMO) wireless system with r receive antennas. Let the vector of complex fading coefficients $\mathbf{h} \triangleq [h_1, h_2, \ldots, h_r]^T \in \mathbb{C}^{r \times 1}$ denote the SIMO channel. The equivalent discrete-time baseband system model after matched filtering is given as,

$$\mathbf{y}(k) = \mathbf{h}x(k) + \eta(k), \quad 1 \leq k \leq N_b \tag{1}$$

where the index k denotes the time instant and $\mathbf{y}(k) \in \mathbb{C}^{r \times 1}, x(k) \in \mathbb{C}$ denote the k^{th} received symbol vector and transmitted symbol respectively. The quantity N_b denotes the block length and the vector $\eta(k) \in \mathbb{C}^{r \times 1}$ is spatio-temporally uncorrelated additive white Gaussian noise of power σ_n^2, i.e. $\mathrm{E}\left\{\eta(k)\eta(l)^H\right\} = \sigma_n^2\, \delta(k-l)\mathbf{I}_r$, where $\delta(k) = 1$ if $k = 0$ and 0 otherwise. Let $X_p \in \mathbb{C}^{1 \times L_p}$

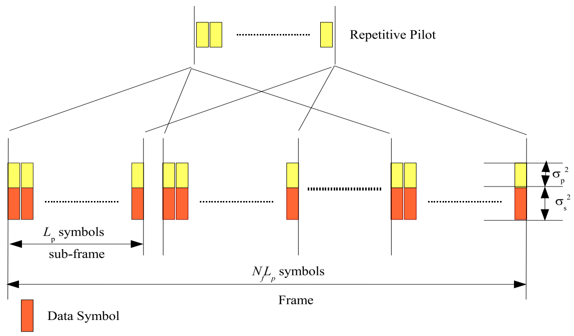

Figure 1: Superimposed pilots frame (block) structure.

defined as, $X_p \triangleq [x_p(1), x_p(2), \ldots, x_p(L_p)]$, be a length L_p symbol pilot sequence of power P_t, i.e. $X_p X_p^H = P_t L_p$. The transmitted data symbols denoted as $x_d(k)$ are assumed to be stochastic in nature with $\mathrm{E}\{x_d(k)\} = 0$ and power $P_d = \mathrm{E}\left\{|x_d(k)|^2\right\}$. Also, let $\rho_d \triangleq \left(P_d/\sigma_n^2\right)$ and $\rho_t \triangleq \left(P_t/\sigma_n^2\right)$ be the signal-to-noise power ratio (SNR) and pilot-to-noise power ratio (PNR) respectively. Below we describe the competing schemes for pilot symbol based channel estimation of the wireless channel $\mathbf{h}$.

2.1. Superimposed pilots (SP) based channel estimation

For SP estimation let each frame of contiguous transmitted symbols contain N_f sub-frames of length L_p symbols, thus making the frame consist of $N_b \triangleq N_f L_p$ symbols. Let $X_d^s \triangleq [x_d^s(1), x_d^s(2), \ldots, x_d^s(N_b)] \in \mathbb{C}^{1\times N_b}$ be the transmitted information symbol sequence of power P_d^s, i.e. $\mathrm{E}\left\{|x_d^s(k)|^2\right\} = P_d^s$. Each such sub-frame consists of independent data symbols with the pilot sequence X_p of L_p symbols superimposed over the data symbols, i.e. $x_p^s(k) = x_p\left(mod(k, L_p) + 1\right)$. A schematic diagram of this SP frame structure is given in fig. 1. The actual transmitted symbol at the k^{th} instant, $x^s(k)$, is therefore given as $x^s(k) \triangleq x_d^s(k) + x_p^s(k)$. The SP system model can be denoted as,

$$\mathbf{y}^s(k) = \mathbf{h}\underbrace{\left(x_d(k) + x_p\left(mod(k, L_p) + 1\right)\right)}_{x^s(k)} + \eta(k), \quad (2)$$

where $\mathbf{y}^s(k), x^s(k)$ are the k^{th} received symbol vector and transmitted symbol respectively. We employ a scheme similar to the ones suggested in [1, 6] to estimate the channel vector $\mathbf{h}$, which is described as follows. Let $\bar{\mathbf{y}}^s(k) \in \mathbb{C}^{r\times 1}, 1 \leq k \leq L_p$ be, $\bar{\mathbf{y}}^s(k) \triangleq \frac{1}{N_f}\sum_{j=0}^{N_f-1} \mathbf{y}^s\left(k + jL_p\right)$. Let $\bar{Y}^s \in \mathbb{C}^{r\times L_p} \triangleq [\bar{\mathbf{y}}^s(1), \bar{\mathbf{y}}^s(2), \ldots, \bar{\mathbf{y}}^s(L_p)]$, be a stacking of the received symbol vectors. Statistically $\mathrm{E}\left\{\bar{Y}^s\right\} = \mathbf{h}X_p$. The channel estimate $\hat{\mathbf{h}}_s$ is now computed by the standard least squares procedure as,

$$\hat{\mathbf{h}}_s = \bar{Y}_s X_p^\dagger = \bar{Y}_s X_p^H \left(X_p X_p^H\right)^{-1}. \quad (3)$$

where $\dagger$ denotes the pseudo-inverse. We refer to the above estimate as the *mean-estimate* for superimposed pilots. This scheme has the advantage of depending only on the first order statistics (mean of received signal $\mathbf{y}^s(k)$) and converges faster (compared to second and higher order statistics based methods) while having a low complexity of implementation. The estimate $\hat{\mathbf{h}}_s$ is then used for detection of the transmitted data $x_d^s(k)$ after subtracting the superimposed pilot symbol.

2.2. Conventional Pilots (CP) based estimation

The CP symbol frame consists of a transmission of one sub-frame, i.e. L_p pilot symbols followed by $(N_f - 1)\, L_p$ information bearing data symbols. To transmit equal total power as in SP, we scale the CP pilot and data powers as $P_t^c = P_t^s N_f$, $P_d^c = P_d^s / \left(1 - \frac{1}{N_f}\right)$. Thus, the CP pilot symbol matrix X_p^c is given as $X_p^c = \sqrt{N_f} X_p$. The input-output model for the CP system is given as, $\mathbf{y}^c(k) = \mathbf{h}x^c(k) + \eta(k)$, where

$$x^c(k) = \begin{cases} \sqrt{N_f}\, x_p^s(k), & k \leq L_p \\ x_d^s(k)/\sqrt{1 - \frac{1}{N_f}}, & k > L_p \end{cases}$$

Defining a stacking of the received pilot symbol outputs as $Y_p^c \triangleq [\mathbf{y}^c(1), \mathbf{y}^c(2), \ldots, \mathbf{y}^c(L_p)]$, the conventional estimate $\hat{\mathbf{h}}_c$ is then given by the well known LS estimate as, $\hat{\mathbf{h}}_c = Y_p^c \left(X_p^c\right)^\dagger$. In the next section we derive analytical expressions for the MSE performance of the estimation schemes presented above.

3. MSE OF ESTIMATION: SP AND CP

The quantity $\bar{Y}^s$ is given as, $\bar{Y}^s = \mathbf{h}X_p + \mathbf{h}\bar{X}_d^s + \bar{N}$, where $\bar{X}_d^s$ and $\bar{N}$ are defined analogously for $x_d^s(k), \eta(k), 1 \leq k \leq N_b$. Simplifying the expression for the SP estimate given in (3), the quantity $\hat{\mathbf{h}}_s$ can be seen to be given as, $\hat{\mathbf{h}}_s = \mathbf{h} + \frac{1}{L_p P_t^s}\left(\mathbf{h}\bar{X}_d^s X_p^H + \bar{N} X_p^H\right)$. Hence, the MSE of the mean-estimate for SP denoted by MSE_s, is given as,

$$\mathrm{MSE}_s = \mathrm{E}\left\{\left\|\hat{\mathbf{h}}_s - \mathbf{h}\right\|^2\right\} = \frac{1}{N_b P_t^s}\left(\|\mathbf{h}\|^2 P_d^s + r\sigma_n^2\right).$$

The error of the CP estimate denoted by MSE_c is given as, $\mathrm{MSE}_c = \frac{r\sigma_n^2}{N_b P_t^s}$[7]. Thus it can be seen that $\mathrm{MSE}_s > \mathrm{MSE}_c$, proving that CP is more suited for the estimation of static channels. Further, it can also be seen that $\mathrm{MSE}_s = \mathrm{MSE}_s^\infty + \mathrm{o}\left(P_d^s\right)$, where $\mathrm{MSE}_s^\infty \in \mathrm{O}\left(P_d^s\right)$ is the dominant component of the MSE of the mean-estimate at high SNR (i.e. $P_d^s \to \infty$) and is given as,

$$\mathrm{MSE}_s^\infty = P_d^s\left(\lim_{P_d^s\to\infty}\frac{\mathrm{MSE}_s}{P_d^s}\right) = \frac{P_d^s}{N_b P_t^s}\|\mathbf{h}\|^2, \quad (4)$$

where as $\mathrm{MSE}_c^\infty = 0$ (defined similarly). Hence MSE_s can be seen to increase progressively increase without bound as the source power P_d^s increases. Thus, increasing P_d^s causes degradation of the estimate $\hat{\mathbf{h}}_s$, which can potentially result in poor detection performance. Next, we derive the Cramer-Rao Bound (CRB) for SP based estimation.

R	SP	CP
$\mathbf{R}_{vs}$	$\mathbf{R}_{vs}^s = -\frac{(P_d^s)^2}{N_b P_t^s}\left(1+\frac{1}{N_b}\right)\mathbf{h}\mathbf{h}^H - \frac{\sigma_n^2 P_d^s}{N_b P_t^s}\mathbf{I}_r$	$\mathbf{R}_{vs}^c = -\frac{\sigma_n^2 P_d^c}{N_f P_t^c}\mathbf{I}_r$
$\mathbf{R}_s$	$\mathbf{R}_s^s = P_d^s\left(1+\frac{P_d^s}{N_b P_t^s}\right)\mathbf{h}\mathbf{h}^H + \frac{\sigma_n^2 P_d^s}{N_b P_t^s}\mathbf{I}_r$	$\mathbf{R}_s^c = P_d^c\mathbf{h}\mathbf{h}^H + \frac{P_d^c\sigma_n^2}{N_f P_t^c}\mathbf{I}_r$
$\mathbf{R}_v$	$\mathbf{R}_v^s = \sigma_n^2\mathbf{I}_r + (P_d^s + P_t^s)\left(\frac{P_d^s\mathbf{h}\mathbf{h}^H}{N_b P_t^s} + \frac{\sigma_n^2\mathbf{I}_r}{N_b P_t^s}\right)$	$\mathbf{R}_v^c = \sigma_n^2\mathbf{I}_r + \frac{\sigma_n^2 P_d^c}{N_f P_t^c}\mathbf{I}_r$

Table 1: Table showing covariance matrices for SP and CP based systems with channel estimation error.

3.1. Cramer-Rao Bound (CRB) for SP Estimation

In this section, we compute the complex CRB for the SP based estimation of $\mathbf{h}$. We assume a Gaussian symbol source i.e. $x_d^s(k) \sim \mathcal{N}(0, P_d^s)$. As suggested in [8] for the construction of CRBs for complex parameters, let the complex parameter vector $\bar{\theta} \in \mathbb{C}^{2r\times 1}$ be constructed by stacking the parameter vector $\mathbf{h}$ and its conjugate as $\bar{\theta} = \left[\mathbf{h}^T, \mathbf{h}^H\right]^T$. From the SP system model for pilot symbol outputs given in (2), the parameter dependent log-likelihood $\mathcal{L}\left(Y^s | X_p^s; \bar{\theta}\right)$ (log-likelihood ignoring additive constants) for the estimation of the parameter vector $\bar{\theta}$ is given as,

$$-N_b \ln|\mathbf{R}_e| - \sum_{i=1}^{N_b}\left(\mathbf{y}^s(i) - \mathbf{h}x_p^s(i)\right)^H \mathbf{R}_e^{-1}\left(\mathbf{y}^s(i) - \mathbf{h}x_p^s(i)\right)$$

where $Y^s \triangleq [\mathbf{y}^s(1), \mathbf{y}^s(2), \ldots, \mathbf{y}^s(N_b)]$ and $\mathbf{R}_e$, the covariance of this effective noise is given as $\mathbf{R}_e \triangleq P_d^s\mathbf{h}\mathbf{h}^H + \sigma_n^2\mathbf{I}_r$. The Cramer-Rao Bound (CRB) for the estimation of $\bar{\theta}$ is given by the matrix $J_{\bar{\theta}}^{-1}$, where $J_{\bar{\theta}} \in \mathbb{C}^{2r\times 2r}$ is the complex Fisher information matrix (FIM) for the parameter vector $\bar{\theta} \in \mathbb{C}^{2r\times 1}$ and is given as $J_{\bar{\theta}} = J_{\bar{\theta}}^p + J_{\bar{\theta}}^r$ where the Pilot FIM (PFIM) component $J_{\bar{\theta}}^p$ of the total FIM $J_{\bar{\theta}}$ is,

$$J_{\bar{\theta}}^p = (N_b P_t^s)\begin{bmatrix} \left(\mathbf{R}_e^{-1}\right)^T & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_e^{-1} \end{bmatrix}.$$

and the FIM component $J_{\bar{\theta}}^r$, which corresponds to the information in the covariance matrix $\mathbf{R}_e$, is given as

$$J_{\bar{\theta}}^r = \frac{N_b\left(P_d^s\right)^2}{\sigma_n^2 + P_d^s\|\mathbf{h}\|^2}\begin{bmatrix} \|\mathbf{h}\|^2\left(\mathbf{R}_e^{-1}\right)^T & \frac{\mathbf{h}^*\mathbf{h}^H}{\sigma_n^2 + P_d^s\|\mathbf{h}\|^2} \\ \frac{\mathbf{h}\mathbf{h}^T}{\sigma_n^2 + P_d^s\|\mathbf{h}\|^2} & \|\mathbf{h}\|^2\mathbf{R}_e^{-1} \end{bmatrix}.$$

The expressions for the FIM components $J_{\bar{\theta}}^p$, $J_{\bar{\theta}}^r$ can be employed to obtain the true FIM $J_{\bar{\theta}}$ and the MSE bound $\mathrm{MSE}_b = \frac{1}{2}\mathrm{tr}\left(J_{\bar{\theta}}^{-1}\right)$. The result below yields a critical insight into the relation between this MSE bound MSE_b and the quantities $\mathrm{MSE}_s, \mathrm{MSE}_c$.

Theorem 1. *The MSE bound for SP based estimation is given as* $\mathrm{MSE}_b = \mathrm{MSE}_b^\infty + \mathrm{o}\left(\mathrm{P_d^s}\right)$ *where* $\mathrm{MSE}_b^\infty \in \mathrm{O}\left(P_d^s\right)$ *is the dominant component of the MSE bound at high SNR ($P_d^s \to \infty$) and is given as,*

$$\mathrm{MSE}_b^\infty = P_d^s\left(\lim_{P_d^s\to\infty}\frac{\mathrm{MSE}_b}{P_d^s}\right) = \frac{1}{2}\left(\frac{P_d^s}{N_b P_t^s}\right)\|\mathbf{h}\|^2. \quad (5)$$

Proof. Given in [5]. □

The quantity MSE_b^∞ also increases linearly with P_d as against $\mathrm{MSE}_c^\infty = 0$. Hence CP outperforms the best SP estimate, and therefore every SP estimate, for a reasonably high SNR. We also have the following result.

Lemma 1. *The asymptotic MSE measures for SP based estimation,* MSE_b^∞ *and* MSE_s^∞, *the asymptotic* MSE *bound and the asymptotic* MSE *of the mean-estimate respectively, are related as,*

$$\frac{\mathrm{MSE}_b^\infty}{\mathrm{MSE}_s^\infty} = \frac{1}{2}. \quad (6)$$

Proof. Follows from (4) and (5). □

Hence, neglecting the covariance information in $\mathbf{R}_e$ results in a 3 dB loss of estimation performance by the mean-estimator. We now describe a semi-blind scheme that achieves the CRB. One can estimate $\tilde{\mathbf{h}}$ from an eigen decomposition of $\mathbf{R}_e$ such that $\mathbf{h} = \tilde{\mathbf{h}}e^{j\hat{\phi}_b}$. It can then be demonstrated that the optimal phase $e^{j\hat{\phi}_b}$ and the semi-blind estimate $\hat{\mathbf{h}}_b$ are,

$$\hat{\mathbf{h}}_b = \tilde{\mathbf{h}}e^{j\hat{\phi}_b} \quad \text{where} \quad \hat{\phi}_b = -\angle\left\{\mathrm{tr}\left(\left(Y^s\right)^H\mathbf{R}_e^{-1}\tilde{\mathbf{h}}X_p^s\right)\right\}.$$

The '$\angle$' operator above yields the angle or phase of the complex scalar quantity. This is akin to the whitening-rotation semi-blind procedure elaborated in [8] and achieves the MSE lower bound in (5) at high SNR.

4. THROUGHPUT COMPARISON

One of the promising aspects of SP based estimation schemes is the potential savings in bandwidth due to the transmission of superimposed data and pilot signals. The result in [9] provides a succinct expression to characterize the worst case capacity of a communication channel in the presence of channel estimation errors. This framework relies on the central assumption that the channel estimate $\hat{\mathbf{h}}$ and the estimation error $\mathbf{h} - \hat{\mathbf{h}}$ satisfy the decorrelation property, i.e. $\mathrm{E}\left\{\hat{\mathbf{h}}\left(\mathbf{h} - \hat{\mathbf{h}}\right)^H\right\} = \mathbf{0}_{r\times r}$, which is satisfied by the minimum mean-squared error (MMSE) estimate. However, this decorrelation property is not satisfied by the least-squares (LS) estimator, which is a disadvantage since the LS estimator is robust and has a low computational complexity, which makes it especially suited for implementation in wireless systems. Further, this result cannot be used in the context

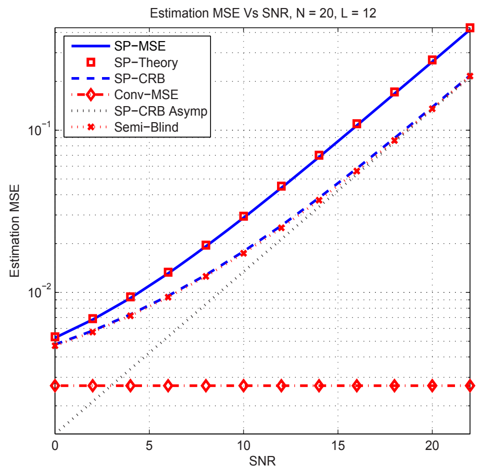

Figure 2: MSE of Estimation of $\mathbf{h}$ with $r = 4$ antennas. N_f = 20, L_p = 8, PNR(ρ_t) = 5dB.

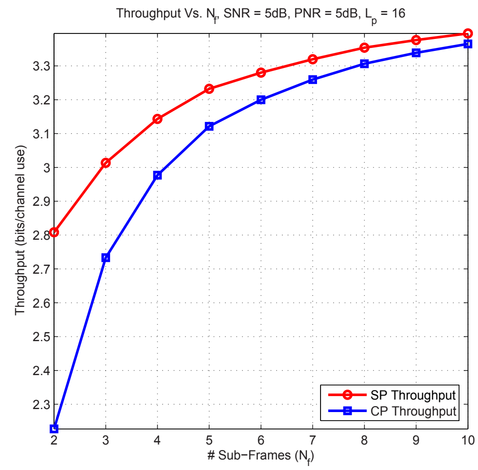

Figure 3: Throughput performance Vs. N_f

of SP based estimation since the SP channel estimate $\hat{\mathbf{h}}_s$ is correlated with the data symbols $x_d^s(k)$ as, $\mathrm{E}\left\{\hat{\mathbf{h}}_s x_d^s(k)\right\} = \left(\frac{P_d^s}{N_b P_t^s}\right)\mathbf{h}x_p^s(mod(k, L_p) + 1)$. Hence, we present a result in this section for the worst case capacity C_w of a channel with non-zero signal-noise correlation.

Lemma 2. Worst case correlated capacity: *Let the system input-output model of a vector output noisy communication channel be given as, $\mathbf{y}(k) = \mathbf{s}(k) + \mathbf{v}(k)$, where $\mathbf{s}(k), \mathbf{v}(k) \in \mathbb{C}^{r\times 1}$ represent the signal and the unknown noise components respectively. Let the covariance matrices of $\mathbf{v}(k), \mathbf{s}(k)$ be given by $\mathbf{R}_s$ and $\mathbf{R}_v$ respectively. Further, let the correlation between the signal and noise components be given as, $\mathrm{E}\left\{\mathbf{v}(k)\mathbf{s}(l)^H\right\} = \delta(k-l)\mathbf{R}_{vs} = \delta(k-l)\mathbf{R}_{sv}^H$ where $\mathbf{R}_{vs}$ is not necessarily $\mathbf{0}_{r\times r}$. For the above communication system, the worst case capacity C_w defined as,*

$$C_w = \min_{p_v(\cdot),\, \mathrm{tr}(\mathbf{R}_v)=r\sigma_n^2} \; \max_{p_s(\cdot)} \mathbf{I}(\mathbf{y};\mathbf{s}),$$

is given by the expression,

$$\min_{\mathrm{tr}(\mathbf{R}_v)=r\sigma_n^2} \log\left|\mathbf{I} + \mathbf{R}_{v|s}^{-1}(\mathbf{R}_s + \mathbf{R}_{vs})\mathbf{R}_s^{-1}(\mathbf{R}_s + \mathbf{R}_{vs})^H\right|,$$

where the conditional covariance $\mathbf{R}_{v|s} \in \mathbb{C}^{r\times r}$ is given as $\mathbf{R}_{v|s} \triangleq \mathbf{R}_v - \mathbf{R}_{vs}\mathbf{R}_s^{-1}\mathbf{R}_{sv}$.

Proof. Given in [5]. □

We employ $\mathbf{R}_v = \sigma_n^2\mathbf{I}$ to derive worst case capacities of the SP and CP estimation schemes. Also, it can be seen for the case of uncorrelated noise, i.e. $\mathbf{R}_{sv} = \mathbf{R}_{vs} = \mathbf{0}_{r\times r}$, the expression above reduces to $C_w = \log\left|\mathbf{I} + \mathbf{R}_v^{-1}\mathbf{R}_s\right|$, which is the result in [9].

4.1. Worst Case Throughput for SP and CP Estimation:

Let $\tilde{\mathbf{y}}^s(k)$ denote the output of the SP system after removal of the pilot symbol $x_p(mod(k, L_p) + 1)$ employing the estimate $\hat{\mathbf{h}}_s$. The effective noise $\mathbf{v}^s(k)$ after pilot removal is given as, $\left(\mathbf{h} - \hat{\mathbf{h}}_s\right)(x_d^s(k) + x_p(mod(k, L_p) + 1)) + \eta(k)$, and $\mathbf{s}^s(k) \triangleq \hat{\mathbf{h}}_s x_d^s(k)$ denotes the souce component. The covariance matrices $\mathbf{R}_s, \mathbf{R}_v, \mathbf{R}_{vs}$ for SP and CP based estimation are given in table 1. Substituting the above quantities in the expression for the worst case capacity, one can obtain the throughput lower bounds for SP and CP systems in terms of bits per channel use. As illustrated by the simulation results, for reasonable values of SNR ($= P_d/\sigma_n^2$), PNR($= P_t/\sigma_n^2$) and number of sub-frames($= N_f$), an SP scheme has a throughput of approximately 0.5 bits per channel use greater than that of CP. This is because CP is disadvantaged by the loss of one sub-frame of bandwidth due to the transmission of pilot symbols exclusively, while the estimation errors are comparable at low SNRs.

5. ESTIMATION ERROR AND PSNR

It can be seen that $\hat{\mathbf{h}}_s$, the estimate of the channel is corrupted by the data symbols $x_d^s(k)$ which enhance the noise during the estimation of the channel. This scenario presents an interesting tradeoff in SP systems. While on one hand, higher data power improves the detection performance, it also results in a poor channel estimate and loss in detection performance. In fact, for a given number of frames N_f, if the source power P_d^s is too high, the detection performance tends to be very poor. Motivated by this observation, we derive expressions for the optimal data SNR $\rho_d^s \left(= P_d^s/\sigma_n^2\right)$ to maximize the post-processing SNR (PSNR) for different receive beamformers.

5.1. MVDR Beamformer

The Minimum Variance Distortionless Response (MVDR) beamformer $\mathbf{w}_m$ is given as a solution to the criterion, $\mathbf{w}_m = \arg\min \mathbf{w}^H\mathbf{R}_v^s\mathbf{w}$, subject to,$\mathbf{w}^H\hat{\mathbf{h}}_s = 1$. The vector $\mathbf{w}_m$

is given as, $\mathbf{w}_m^H = \left(\hat{\mathbf{h}}_s^H (\mathbf{R}_v^s)^{-1} \hat{\mathbf{h}}_s\right)^{-1} \hat{\mathbf{h}}^H (\mathbf{R}_v^s)^{-1}$. Substituting this above, the expression for the post-processing SNR of the MVDR beamformer can be seen to be given as,

$$\kappa_m = \frac{P_d^s}{\mathrm{E}\left\{|\mathbf{w}_m^H \mathbf{v}^s(k)|^2\right\}} = P_d^s \hat{\mathbf{h}}_s^H (\mathbf{R}_v^s)^{-1} \hat{\mathbf{h}}_s. \quad (7)$$

As demonstrated in [5], the above expression can be simplified by substituting the expression for $\mathbf{R}_v^s$ in table 1 to yield,

$$\kappa_m \approx \frac{\rho_t^s \rho_d^s N_b \|\mathbf{h}\|^2}{(\rho_d^s + \rho_t^s)\rho_d^s \|\mathbf{h}\|^2 + \rho_t^s (N_b + 1) + \rho_d^s}, \quad (8)$$

where ρ_d^s, ρ_t^s are the data and pilot SNR respectively as defined previously. The optimum ρ_d^{mvdr} that maximizes the above expression for the post-processing SNR κ_m is given by the following result.

Lemma 3. *The optimum data SNR ρ_d^{mvdr} that maximizes the post-processing SNR κ_m for the MVDR beamformer is given as,*

$$\rho_d^{mvdr} \triangleq \frac{P_d^{mvdr}}{\sigma_n^2} = \sqrt{\frac{(N_b + 1)}{\|\mathbf{h}\|^2}} \rho_t^s \quad (9)$$

Proof. Given in [5]. □

The expression for the transmit SNR that maximizes the channel SNR and PSNR for the matched filter beamformer can be obtained similarly [5].

6. SIMULATION RESULTS

We consider CP vs SP based channel estimation of the static wireless channel $\mathbf{h} \in \mathbb{C}^{4\times 1}$ i.e. $r = 4$ receive antennas. The MSE bound from the SP-CRB is also plotted and can be seen to coincide at high-SNR with the asymptotic expression given in (5). As stated in lemma 1, it is seen that at high SNR, the SP MSE bound is 3dB lower than the MSE of the SP mean-estimate. MSE_b, the MSE of the semi-blind estimate $\hat{\mathbf{h}}_b$ is plotted as 'semi-blind' which can be seen to achieve the asymptotic MSE bound for SP. The MSE of the conventional estimate is plotted as 'Conv-MSE' and can be seen to be the lowest amongst all the curves, thus verifying that CP has the lowest MSE of estimation. In Fig.3. we plot the throughput variation of the above system for SP and CP schemes vs. varying SNR. For $L_p = 16$ sub-frames and SNR = PNR = 5dB, the SP system throughput is around 0.5 bits/channel use higher than that of CP. The throughput performance of the CP increases with SNR as expected. In Fig.4. we also plot the SER performance as a function of SNR, and the computed optimal SNR value for different values of sub-frames N_f, pilot length L_p and pilot to noise power ratio (PNR). It can be seen that the theoretically computed optimal SNR from (9) is fairly accurate for optimal performance i.e. minimum SER.

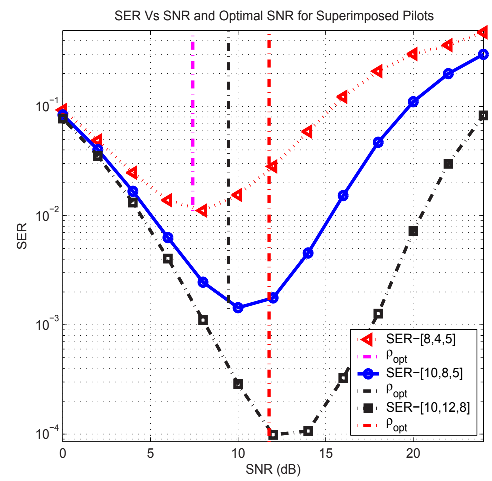

Figure 4: Detection performance vs. SNR of SP based estimation for QPSK signaling, $[N_f, L_p, \rho_t]$.

7. CONCLUSION

We have compared two competing schemes for pilot based channel estimation viz. superimposed pilots (SP) and conventional pilots (CP) employing an MSE and throughput framework. It has been observed that although CP outperforms SP in terms of MSE of estimation, SP can have a higher effective throughput than CP based systems which leads to a savings in bandwidth in communication systems.

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