WR based Semi-Blind Channel Estimation for Frequency-Selective MIMO MC-CDMA Systems

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***Abstract*—In this paper, we propose a novel whitening-rotation (WR) based semi-blind (SB) scheme for frequency-selective channel estimation in multiple-input multiple-output (MIMO) multi-carrier (MC) CDMA systems. This scheme is based on a low complexity multi-path multi-carrier decorrelator (MMD) receiver structure developed for the MIMO MC-CDMA system. It is shown that the MMD based receiver naturally enables the formulation of the SB scheme by reducing the frequency-selective MIMO MC-CDMA channel to a flat fading channel matrix of appropriate dimension. Thus, it results in a significant reduction in the computational complexity usually associated with frequency-selective MIMO channel estimation. Further, we derive the Cramer-Rao bound (CRB) to characterize the mean squared error (MSE) performance of the proposed SB scheme. For performance comparison of the proposed SB estimator, we also derive the training estimate and the uncertainty based robust estimate of the frequency-selective MIMO MC-CDMA channel. Simulation results demonstrate that the proposed SB scheme achieves a significantly lower MSE of estimation compared to the competing training and robust estimation schemes.**

I. Introduction

Third generation (3G) and fourth generation (4G) wireless systems for broadband wireless access have gained rapid popularity in recent years. Multicarrier code division for multiple access (MC-CDMA), which employs a multi-carrier physical layer in conjunction with CDMA, has attracted substantial research interest. MC-CDMA aims at harnessing the advantages of CDMA and orthogonal frequency division multiplexing (OFDM) [1]. Multiple-input multiple-output (MIMO) technology can be employed in MC-CDMA systems to further increase the throughput through spatial multiplexing. However, to achieve maximum performance gains, it is essential to estimate the wireless channel coefficients accurately. For better channel estimation, the conventional training based schemes require transmitting of longer training sequences which results in high communication overhead [2]. This problem can be alleviated by employing superimposed training-based schemes which add a training sequence to the data symbols [3]. However, such schemes result in increased transmit power. On the other hand, blind channel estimation schemes are often computationally complex and estimate the channel only up to phase ambiguities [4]. Hence, semi-blind techniques which utilize the statistical properties of the information symbols to enhance the accuracy of the channel estimate are ideally suited for such scenarios.

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In this context, enabled by a novel multi-path multi-carrier decorrelator (MMD) receiver structure, we propose a whitening-rotation (WR) channel matrix decomposition based semi-blind (SB) algorithm for the estimation of the frequency-selective MIMO MC-CDMA channel. The proposed WR scheme is based on estimating the whitening matrix employing exclusively blind statistical information followed by the rotation uncertainty resolution using a significantly small pilot overhead [5]. Further, we derive the Cramer-Rao Bound (CRB) of the semi-blind MIMO channel estimator. The proposed SB scheme is seen to have a significantly lower mean squared-error (MSE) of estimation compared to the competing training and robust approaches for MIMO MC-CDMA channel estimation. The rest of the paper is organized as follows. In section II we describe the frequency-selective MIMO MC-CDMA system model followed by an outline of the MMD receiver in section III. Section IV describes the proposed WR based SB estimation scheme and the associated CRB. In section V we present the competing training and uncertainty set based robust approaches for MIMO MC-CDMA channel estimation. Simulation results are given in section VI and we present our conclusions with section VII.

II. System Model

Consider a downlink (DL) MIMO MC-CDMA system with N_r receive antennas and N_t transmit antennas. The base-band model of the frequency-selective MIMO MC-CDMA system is given as,

$$\mathbf{y}(n) = \sum_{i=0}^{L_h-1} \mathbf{H}(i)\mathbf{s}(n-i) + \mathbf{v}(n) \tag{1}$$

where $\mathbf{y}(n) \in \mathbb{C}^{N_r \times 1}$ is the received signal at time instant n, $\mathbf{s}(n) \in \mathbb{C}^{N_t \times 1}$ is the composite DL transmit vector at time instant n, $\mathbf{v}(n) \in \mathbb{C}^{N_r \times 1}$ is additive spatio-temporally white Gaussian noise with covariance $\mathrm{E}\left\{\mathbf{v}(n)\mathbf{v}(n)^H\right\} = \sigma_n^2 \mathbf{I}_{N_r}$. Each $\mathbf{H}(i) \in \mathbb{C}^{N_r \times N_t}$, $0 \leq i \leq L_h - 1$, is the channel matrix corresponding to the i^{th} lag and L_h is the length of the MIMO frequency-selective finite impulse response (FIR) channel. Each complex element $h_{r,t}(i)$ of the matrix $\mathbf{H}(i)$ denotes the

$$
\begin{aligned}
d_{r,l}(p) &= \frac{1}{N}\sum_{k=0}^{K-1}\sum_{m=0}^{N-1}\sum_{t=1}^{N_t}\sum_{i=0}^{L_h-1} a_{k,t}(p)c_{k,m}c_{0,m}^{*}h_{r,t}(i)e^{-j2\pi(i-l)\frac{m}{N}} + \tilde{v}(p) \\
&= \sum_{t=1}^{N_t} h_{r,t}(l)a_{0,t}(p) + \underbrace{\frac{1}{N}\sum_{t=1}^{N_t}\sum_{\substack{i=0\\ i\neq l}}^{L_h-1}\sum_{m=0}^{N-1} a_{0,t}(p)h_{r,t}(i)e^{-j2\pi(i-l)\frac{m}{N}}}_{d_{\text{ISI}}} \\
&\quad + \underbrace{\frac{1}{N}\sum_{k=1}^{K-1}\sum_{t=1}^{N_t}\sum_{m=0}^{N-1}\sum_{i=0}^{L_h-1} a_{k,t}(p)c_{k,m}c_{0,m}^{*}h_{r,t}(i)e^{-j2\pi(i-l)\frac{m}{N}}}_{d_{\text{MUI}}} + \underbrace{\frac{1}{N}\sum_{m=0}^{N-1} V_{r,m}(p)c_{0,m}^{*}e^{j2\pi l\frac{m}{N}}}_{\tilde{v}(p)}
\end{aligned} \tag{4}
$$

---

channel coefficient between transmit antenna t and receive antenna r corresponding to the i^{th} delay. Let K denote the total number of DL users of the MIMO MC-CDMA system. Let the symbol vector of k^{th} user, transmitted in the p^{th} MC block, be denoted by $\mathbf{a}_k(p) = [a_{k,1}(p), a_{k,2}(p), \ldots, a_{k,N_t}(p)]^T$, where $a_{k,t}(p)$ is the symbol corresponding the t^{th} transmit antenna. Let the covariance of the transmit symbol vector $\mathbf{a}_k(p)$ be given as $\mathrm{E}\left\{\mathbf{a}_k(p)\mathbf{a}_k^H(p)\right\} = P_d\mathbf{I}_{N_t}$ where P_d is the transmit power corresponding to each transmit antenna. The DL transmit symbol vector loaded onto the m^{th} subcarrier in this multi-carrier system is given by $\mathbf{X}_m(p) = \sum_{k=0}^{K-1} c_{k,m}\mathbf{a}_k(p)$, where $\mathbf{X}_m(p) \in \mathbb{C}^{N_t\times 1}$, $\{c_{k,m}\}_{m=0}^{N-1}$ is the spreading code of the k^{th} user and the spreading length N is equal to the number of subcarriers. The composite DL signal $\mathbf{s}(n)$ is given by the N-point IFFT of the spread data symbol vectors $\mathbf{X}_m(p)$ followed by the addition of the cyclic prefix. Hence, after serial-to-parallel conversion, removal of cyclic prefix, and N-point FFT at the receiver, $\mathbf{Y}_m(p) \in \mathbb{C}^{N_r\times 1}$, the received data at subcarrier m is given as [6],

$$
\mathbf{Y}_m(p) = \mathbf{Z}_m\mathbf{X}_m(p) + \mathbf{V}_m(p), \tag{2}
$$

where $\mathbf{Z}_m \in \mathbb{C}^{N_r\times N_t}$, the flat-fading channel coefficient matrix corresponding to the m^{th} subcarrier, is given by the N-point FFT of the MIMO frequency-selective channel $\mathbf{H}(i)$, $0 \le i \le L_h - 1$ as,

$$
\mathbf{Z}_m = \sum_{i=0}^{L_h-1} \mathbf{H}(i)e^{-j2\pi i\frac{m}{N}}. \tag{3}
$$

The quantity $\mathbf{V}_m(p)$ is the FFT of the receiver noise vectors $\mathbf{v}(n)$. Below we describe the optimal multi-path multi-carrier decorrelator based semi-blind channel estimator of the frequency selective MIMO MC-CDMA channel, described by the channel taps $\mathbf{H}(i)$, $0 \le i \le L_h - 1$.

III. Multi-path Multi-carrier Decorrelator for MIMO MC-CDMA

From the expression in (2), the r^{th} element of the received data vector $\mathbf{Y}_m(p) \in \mathbb{C}^{N_r\times 1}$, corresponding to the received symbol at the r^{th} antenna on the m^{th} subcarrier can be readily seen to be given as,

$$
\begin{aligned}
Y_{r,m}(p) &= \sum_{k=0}^{K-1}\sum_{t=1}^{N_t} z_{r,t}^{m}a_{k,t}(p)c_{k,m} + V_{r,m}(p), \\
&= \sum_{k=0}^{K-1}\sum_{t=1}^{N_t}\sum_{i=0}^{L_h-1} h_{r,t}(i)e^{-j2\pi i\frac{m}{N}}a_{k,t}(p)c_{k,m} + V_{r,m}(p),
\end{aligned} \tag{5}
$$

where $V_{r,m}(p)$ is the noise at receive antenna r on subcarrier m. Equation (5) follows by observing from (3) that $z_{r,t}^{m} = \sum_{i=0}^{L_h-1} h_{r,t}(i)e^{-j2\pi i\frac{m}{N}}$, where $z_{r,t}^{m}$ and $h_{r,t}(i)$ are the (r,t) elements of the matrices $\mathbf{Z}_m$ and $\mathbf{H}(i)$ respectively. Consider the signal detection at the 0^{th} user, with the remaining $K-1$ $(1 \le k \le K-1)$ users considered as interferers. The received data $Y_{r,m}(p)$ at user 0 can be decorrelated with the spreading code $\{c_{0,m}\}_{m=0}^{N-1}$ to obtain the statistic $d_{r,l}(p)$ corresponding to receive antenna r and delay l. However, in MC-CDMA systems, the optimal decorrelator has to be compensated for the carrier offset arising from the delay of the l^{th} path, $0 \le l \le L_h - 1$. Hence, the optimal multi-path multi-carrier decorrelation (MMD) statistic $d_{r,l}(p)$ is given as,

$$
d_{r,l}(p) = \frac{1}{N}\sum_{m=0}^{N-1} Y_{r,m}(p)c_{0,m}^{*}e^{j2\pi l\frac{m}{N}}. \tag{6}
$$

The above MMD statistic naturally exploits the correlation properties of the spreading code to leverage multi-path diversity, akin to the rake receiver in single carrier CDMA systems [7], [8]. Substituting the expression for $Y_{r,m}(p)$ from (5), the MMD statistic $d_{r,l}(p)$ can be expanded as in (4). Hence, the equivalent system model for the reception of the transmit symbol vector $\mathbf{a}_0(p)$ in the presence of the inter symbol interference (ISI) and multi-user interference (MUI) components, $d_{\text{ISI}}(p)$ and $d_{\text{MUI}}(p)$ respectively, is given as

$$
d_{r,l}(p) = \mathbf{h}_r^T(l)\mathbf{a}_0(p) + d_{\text{ISI}}(p) + d_{\text{MUI}}(p) + \tilde{v}(p), \tag{7}
$$

where the channel coefficient vector $\mathbf{h}_r^T(l)$ corresponds to elements of the r^{th} row of the matrix $\mathbf{H}(l)$, and is defined as $\mathbf{h}_r(l) = [h_{r,1}(l), h_{r,2}(l), \ldots, h_{r,N_t}(l)]^T$. It can also be seen that $\tilde{v}(p)$ is the effective additive Gaussian noise at the output of the MMD despreader with $\sigma_{\tilde{v}}^2 = \mathrm{E}\left\{\tilde{v}(p)\tilde{v}^*(p)\right\} = \frac{\sigma_n^2}{N}$. Further, employing the orthogonality properties of the subcarriers, it can be readily seen that,

$$d_{\text{ISI}}(p) = \frac{1}{N}\sum_{t=1}^{N_t}\sum_{\substack{i=0\\ i\neq l}}^{L_h-1} a_{0,t}(p)h_{r,t}(i)\left[\sum_{m=0}^{N-1} e^{-j2\pi(i-l)\frac{m}{N}}\right] = 0, \tag{8}$$

where the last equality follows from the observation that $\frac{1}{N}\sum_{n=0}^{N-1} e^{-j2\pi(i-l)\frac{n}{N}} = 0$ when $i \neq l$. Further $d_{\text{MUI}}(p) \to 0$ for large spreading lengths N and is negligible. Hence, the system model (7) can be recast as,

$$d_{r,l}(p) = \mathbf{h}_r^T(l)\mathbf{a}_0(p) + \tilde{v}(p). \tag{9}$$

The net MMD decision statistic $\mathbf{d}_l(p)$ corresponding to the l^{th} lag is given by the vector $\mathbf{d}_l(p) \triangleq [d_{1,l}(p), d_{2,l}(p), \ldots, d_{N_r,l}(p)]^T$. Stacking the decision statistics for the L_h lags, $0 \leq l \leq L_h-1$, the N_rL_h dimensional decision statistic for the frequency-selective MIMO MC-CDMA system model is given as,

$$\begin{aligned}\mathbf{d}(p) &\triangleq \left[\mathbf{d}_0^T(p), \mathbf{d}_1^T(p), \ldots, \mathbf{d}_{L_h-1}^T(p)\right]^T \\ &= \boldsymbol{\mathcal{H}}\mathbf{a}_0(p) + \widetilde{\mathbf{v}}(p),\end{aligned} \tag{10}$$

where the block matrix $\boldsymbol{\mathcal{H}} \in \mathbb{C}^{N_rL_h\times N_t}$ is defined as

$$\boldsymbol{\mathcal{H}} \triangleq \left[\mathbf{H}^T(0), \mathbf{H}^T(1), \ldots, \mathbf{H}^T(L_h-1)\right]^T. \tag{11}$$

The noise vector $\widetilde{\mathbf{v}}(p) \in \mathbb{C}^{N_rL_h\times 1}$ is Gaussian with covariance $\mathrm{E}\{\widetilde{\mathbf{v}}(p)\widetilde{\mathbf{v}}^H(p)\} = \frac{\sigma_n^2}{N}\mathbf{I}_{N_rL_h}$. Thus, the MMD based MIMO MC-CDMA system model above reduces the complexity of estimation of the L_h lag frequency-selective MIMO MC-CDMA channel to that of an equivalent $N_rL_h \times N_t$ flat-fading MIMO channel. Hence, in contrast to commonly employed approaches such as [9], which involve the individual estimation of channel matrices corresponding to different lags, the above procedure is equivalent to a single stage flat-flading channel estimation. In the next section, we explain the whitening-rotation (WR) based semi-blind (SB) scheme for the estimation of the frequency-selective MIMO MC-CDMA channel.

IV. Semi-Blind Channel Estimation

As described in section I, semi-blind (SB) schemes are ideally suited for channel estimation in wireless communication systems since they significantly reduce the pilot overhead, while resolving the parameter uncertainties and avoiding the convergence problems associated with blind channel estimation algorithms. A novel SB scheme for estimation of the channel matrix $\boldsymbol{\mathcal{H}} \in \mathbb{C}^{N_rL_h\times N_t}$ at the output of the MMD receiver described above for the frequency-selective MIMO MC-CDMA channel, can be obtained by decomposing $\boldsymbol{\mathcal{H}}$ as $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{Q}}^H$, where $\boldsymbol{\mathcal{W}} \in \mathbb{C}^{N_rL_h\times N_t}$ is the whitening matrix and $\boldsymbol{\mathcal{Q}} \in \mathbb{C}^{N_t\times N_t}$ is the unitary rotation matrix. For instance, if the singular value decomposition (SVD) of $\boldsymbol{\mathcal{H}}$ is given as $\boldsymbol{\mathcal{H}} = \boldsymbol{\mathcal{U}}\boldsymbol{\Sigma}\boldsymbol{\mathcal{V}}^H$, the whitening matrix $\boldsymbol{\mathcal{W}}$ can be expressed as $\boldsymbol{\mathcal{W}} = \boldsymbol{\mathcal{U}}\boldsymbol{\Sigma}$ and the rotation matrix as $\boldsymbol{\mathcal{Q}} = \boldsymbol{\mathcal{V}}$. Since the unitary matrix $\boldsymbol{\mathcal{Q}}$ is constrained as $\boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{Q}}^H = \boldsymbol{\mathcal{Q}}^H\boldsymbol{\mathcal{Q}} = \mathbf{I}_{N_t}$, significantly fewer parameters are required to describe $\boldsymbol{\mathcal{Q}}$. The whitening matrix $\boldsymbol{\mathcal{W}}$ can be estimated using the second-order statistics of the received data $\mathbf{d}^b(p)$ corresponding to the blind information symbol vectors $\mathbf{a}^b(p)$, $1 \leq p \leq N_b$. It can be readily seen from the model in (10) that the covariance $\mathbf{R_d}$ of the MMD vectors $\mathbf{d}^b(p)$ is given as,

$$\begin{aligned}\mathbf{R_d} &= \boldsymbol{\mathcal{H}}\,\mathrm{E}\left\{\mathbf{a}^b(p)\left(\mathbf{a}^b(p)\right)^H\right\}\boldsymbol{\mathcal{H}}^H + \sigma_{\tilde{v}}^2\mathbf{I} \\ &= P_d\boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{W}}^H + \sigma_{\tilde{v}}^2\mathbf{I},\end{aligned} \tag{12}$$

where the last equality follows from $\mathrm{E}\{\mathbf{a}^b(p)\left(\mathbf{a}^b(p)\right)^H\} = P_d\mathbf{I}_{N_t}$ and $\boldsymbol{\mathcal{H}}\boldsymbol{\mathcal{H}}^H = \boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{Q}}^H\boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{W}}^H = \boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{W}}^H$. For large block lengths, the number of blind information symbols $N_b \to \infty$. Hence, the whitening matrix $\boldsymbol{\mathcal{W}}$ can be estimated to a high degree of accuracy from the N_t dominant left singular vectors corresponding to the estimate of the covariance matrix $\widehat{\mathbf{R}}_d = \frac{1}{N_b}\sum_{p=1}^{N_b}\mathbf{d}^b(p)(\mathbf{d}^b(p))^H$ as,

$$\boldsymbol{\mathcal{W}} = \boldsymbol{\mathcal{U}}_{\mathcal{W}}\boldsymbol{\Sigma}_{\mathcal{W}}^{\frac{1}{2}} \tag{13}$$

$$\text{where } \boldsymbol{\mathcal{U}}_{\mathcal{W}}\boldsymbol{\Sigma}_{\mathcal{W}}\boldsymbol{\mathcal{V}}_{\mathcal{W}}^H = \mathrm{SVD}\left(\frac{1}{P_d}\left[\widehat{\mathbf{R}}_d - \sigma_{\tilde{v}}^2\mathbf{I}\right]\right).$$

The unitary matrix $\boldsymbol{\mathcal{Q}}$ can be estimated employing pilot symbols. Let the matrix $\mathbf{A}_p \in \mathbb{C}^{N_t\times L_p}$, the pilot symbol matrix corresponding to the L_p pilot transmissions $\mathbf{A}_p = [\mathbf{a}(1), \mathbf{a}(2), ..., \mathbf{a}(L_p)]$, be such that $\mathbf{A}_p\mathbf{A}_p^H = L_pP_t\mathbf{I}$, where P_t is training power. The MMD output matrix $\mathbf{D}_p = [\mathbf{d}(1), \mathbf{d}(2), ..., \mathbf{d}(L_p)]$, where $\mathbf{D}_p \in \mathbb{C}^{N_r\times L_p}$, corresponding to the transmission of the L_P pilot symbols is given from (10) as,

$$\mathbf{D}_p = \boldsymbol{\mathcal{H}}\mathbf{A}_p + \widetilde{\mathbf{V}}_p, \tag{14}$$

where the matrix $\widetilde{\mathbf{V}}_p \in \mathbb{C}^{N_r\times L_p}$ corresponds to the noise at the receiver. Hence, the ML estimate $\widehat{\boldsymbol{\mathcal{Q}}}$ of the unitary matrix $\boldsymbol{\mathcal{Q}}$ is obtained by minimizing the constrained likelihood cost function,

$$\begin{aligned}\min_{\boldsymbol{\mathcal{Q}}}.\quad & \|\mathbf{D}_p - \boldsymbol{\mathcal{W}}\boldsymbol{\mathcal{Q}}^H\mathbf{A}_p\|_F^2, \\ \text{s.t.}\quad & \boldsymbol{\mathcal{Q}}\boldsymbol{\mathcal{Q}}^H = \mathbf{I},\end{aligned} \tag{15}$$

where $\|\cdot\|_F$ denotes the matrix Frobenius norm. The closed form expression for $\widehat{\boldsymbol{\mathcal{Q}}}$, which minimizes the above cost function, is given as,

$$\widehat{\boldsymbol{\mathcal{Q}}} = \boldsymbol{\mathcal{V}}_{\mathcal{Q}}\boldsymbol{\mathcal{U}}_{\mathcal{Q}}^H \text{ where } \boldsymbol{\mathcal{U}}_{\mathcal{Q}}\boldsymbol{\Sigma}_{\mathcal{Q}}\boldsymbol{\mathcal{V}}_{\mathcal{Q}}^H = \mathrm{SVD}(\boldsymbol{\mathcal{W}}^H\mathbf{D}_p\mathbf{A}_p^H). \tag{16}$$

Hence, $\boldsymbol{\mathcal{H}}_S$ the SB estimate of the channel matrix $\boldsymbol{\mathcal{H}}$ is given as $\widehat{\boldsymbol{\mathcal{H}}}_S = \boldsymbol{\mathcal{W}}\widehat{\boldsymbol{\mathcal{Q}}}^H$. It can be readily seen that this estimate is semi-blind in nature since it employs the second-order statistical blind information for the estimation of $\boldsymbol{\mathcal{W}}$ in conjunction with the pilot symbol information for the estimation of $\boldsymbol{\mathcal{Q}}$. As shown in the next section, the above SB scheme results in a much lower MSE of estimation compared to conventional training based schemes which exclusively employ pilot information, thus discarding the valuable blind statistical information. Further, the existing blind scheme for the estimation of the frequency-selective MIMO channel [10] involves computing the output correlation matrices for different lags followed by multi-step linear prediction, thereby resulting in a substantial increase in the computational complexity. Hence, the MMD based SB estimation scheme described above significantly reduces the complexity involved in the channel estimation of the MC-CDMA MIMO frequency-selective channel by converting it to an equivalent $N_r L_h \times N_t$ dimensional flat-fading MIMO channel.

A. Cramer-Rao Bound (CRB) for MMD based SB MIMO MC-CDMA Channel Estimation

In this section we derive the Cramer-Rao lower bound of the MMD based SB estimate $\widehat{\boldsymbol{\mathcal{H}}}_S$ of the frequency-selective MIMO MC-CDMA channel. The CRB on the covariance of any unbiased estimator $\hat{\theta}$ for the estimation of a vector parameter θ is given as $\mathrm{E}\{(\hat{\theta}-\theta)(\hat{\theta}-\theta)^H\} \geq \mathbf{J}^{-1}(\theta)$, where $\mathbf{J}(\theta)$ is the Fisher information matrix (FIM) corresponding to the observations $\mathbf{f}$, and is defined as,

$$\mathbf{J}(\theta) \triangleq \mathrm{E}\left(\frac{\partial}{\partial\theta}\log l(\theta,\mathbf{f})\right)\left(\frac{\partial}{\partial\theta}\log l(\theta,\mathbf{f})\right)^T,$$

where $l(\theta,\mathbf{f})$ is the likelihood function corresponding to the observations $\mathbf{f}$, parametrized by θ. Hence it can be seen that the lower bound on the MSE of the estimation of θ is given as $\mathrm{E}\{\|\hat{\theta}-\theta\|_2^2\} \geq \mathrm{tr}\left(\mathbf{J}^{-1}(\theta)\right)$. Further from the result in [5] for the estimation of the constrained parameter vector θ, the CRB on the MSE of the constrained SB estimate $\widehat{\boldsymbol{\mathcal{H}}}_S$ can be readily deduced as,

$$\mathrm{MSE}_S = \mathrm{E}\left\{\|\widehat{\boldsymbol{\mathcal{H}}}_S - \boldsymbol{\mathcal{H}}\|_F^2\right\} \geq \frac{\sigma_{\tilde{v}}^2}{2P_t L_p}\mathcal{N}_{\boldsymbol{\mathcal{H}}} = \frac{\sigma_{\tilde{v}}^2 N_t^2}{2P_t L_p}, \quad (17)$$

where the last equality follows from the fact that $N_{\boldsymbol{\mathcal{H}}} = N_t^2$ is required to estimate the channel matrix $\boldsymbol{\mathcal{H}}$. From the above bound on the estimation we can see that the MSE bound on the SB estimate $\widehat{\boldsymbol{\mathcal{H}}}_S$ is proportional to N_t^2, which is the number of parameters corresponding to the constrained unitary matrix $\boldsymbol{\mathcal{Q}}$.

V. Comparison with Training and Robust Channel Estimation

In this section we present a comparison of the MMD based SB frequency-selective MIMO MC-CDMA channel estimator described above with that of the conventional training based least-squares channel estimator [11] and the uncertainty set based robust channel estimator [12]. For this purpose we derive the CRB of training based estimation and the robust estimator for uncertainty set based estimation.

A. Training based Least-Squares (LS) Channel Estimation

From the expression for the pilot symbol output matrix $\mathbf{D}_p$ given in (14) it can be seen that the maximum-likelihood (ML) training estimate $\widehat{\boldsymbol{\mathcal{H}}}_{\mathrm{T}}$ of the frequency-selective MIMO MC-CDMA channel matrix $\boldsymbol{\mathcal{H}}$ is given by the standard LS estimator [11],

$$\widehat{\boldsymbol{\mathcal{H}}}_{\mathrm{T}} = \mathbf{D}_p\mathbf{A}_p^\dagger = \boldsymbol{\mathcal{H}} + \widetilde{\mathbf{V}}\mathbf{A}_p^\dagger = \boldsymbol{\mathcal{H}} + \frac{1}{L_p P_t}\widetilde{\mathbf{V}}\mathbf{A}_p^H, \quad (18)$$

where the last equality follows from the fact that $\mathbf{A}_p^\dagger$, the pseudo-inverse of pilot symbol matrix $\mathbf{A}_p$ is given as $\mathbf{A}_p^\dagger = \mathbf{A}_p^H\left(\mathbf{A}_p\mathbf{A}_p^H\right)^{-1}$ and $\mathbf{A}_p\mathbf{A}_p^H = L_p P_t \mathbf{I}$, corresponding to the optimal pilot matrix $\mathbf{A}_p$ as described in section IV. Hence, the CRB on the MSE, which is achieved by the optimal LS estimator above, can be derived as,

$$\begin{aligned}\mathrm{MSE}_T = \mathrm{E}\{\|\widehat{\boldsymbol{\mathcal{H}}}_{\mathrm{T}} - \boldsymbol{\mathcal{H}}\|_F^2\} &= \frac{1}{L_p^2 P_t^2}\mathrm{E}\left\{\mathrm{tr}(\widetilde{\mathbf{V}}\mathbf{A}_p^H\mathbf{A}_p\widetilde{\mathbf{V}}^H)\right\} \\ &= \frac{N_t N_r L_h \sigma_{\tilde{v}}^2}{L_p P_t}. \quad (19)\end{aligned}$$

Comparing with the above expression for MSE_T with that of MSE_S, the MSE of the semi-blind estimate $\widehat{\boldsymbol{\mathcal{H}}}_S$, given in (17), it can be readily seen that $\mathrm{MSE_T}$ is significantly greater than $\mathrm{MSE_S}$. This follows from the fact that $N_t N_r L_h \gg N_t^2/2$, since $N_r > N_t$ and $L_h \gg 1$ for the wideband MIMO MC-CDMA frequency selective channel. Further, it is interesting to note that the performance gap increases with the number of resolvable channel paths L_h. Hence, the MMD based SB channel estimation scheme results in a significant reduction in the MSE of channel estimation. This follows naturally from the fact that, while the channel matrix $\boldsymbol{\mathcal{H}}$ is characterized by $2N_t N_r L_h$ parameters, all but N_t^2 parameters of the channel matrix $\boldsymbol{\mathcal{H}}$ can be estimated from the blind information symbols. Thus, the training based estimator which does not employ the statistical properties of the blind information symbols is sub-optimal. The MSE performance of the SB and training schemes illustrated in section VI clearly validates the above conclusion.

B. Uncertainty based Robust Channel Estimation

The robust channel estimator for a single-input single-output (SISO) MC-CDMA system based on the spherical uncertainty set is given in [13]. However, the framework presented therein is restrictive and can not be extended to the frequency-selective MIMO MC-CDMA scenario under consideration. The proposed MMD based channel estimation framework readily lends itself to the paradigm of robust channel estimation. From the discussion in [12], $\widehat{\boldsymbol{\mathcal{H}}}_R$, the uncertainty set based robust estimate of the channel matrix $\boldsymbol{\mathcal{H}}$ at the output of the MMD MIMO MC-CDMA receiver, is given as the solution of the optimization problem,

$$\begin{aligned}\min_{\boldsymbol{\mathcal{H}}_R}.\quad & tr\left(\boldsymbol{\mathcal{H}}_R^H\mathbf{R}_d^{-1}\boldsymbol{\mathcal{H}}_R\right) \\ \text{s.t.}\quad & \|\widehat{\boldsymbol{\mathcal{H}}}_T - \boldsymbol{\mathcal{H}}_R\|_F^2 \leq \epsilon,\end{aligned} \quad (20)$$

where $\mathbf{R_d}$ is given by (12) and $\widehat{\boldsymbol{\mathcal{H}}}_T$ is the least squares estimate of $\boldsymbol{\mathcal{H}}$, as derived above. Further, the expression for the radius of the uncertainty set ϵ can be derived from the above discussion in section V-A as $\epsilon = \frac{N_t N_r L_h \sigma_{\tilde{v}}^2}{L_p P_t}$. Due to lack of space, we avoid an elaborate derivation of the optimal robust estimate $\widehat{\boldsymbol{\mathcal{H}}}_R$ and it can be given as,

$$\widehat{\boldsymbol{\mathcal{H}}}_R = \left(\mathbf{I} - (\mathbf{I} + \lambda\mathbf{R}_d)^{-1}\right)\widehat{\boldsymbol{\mathcal{H}}}_T, \quad (21)$$

where λ is derived as the solution of $\|\left(\mathbf{I}+\lambda\mathbf{R}_d\right)^{-1}\widehat{\boldsymbol{\mathcal{H}}}_T\|_F^2 = \epsilon$, similar to the estimate of

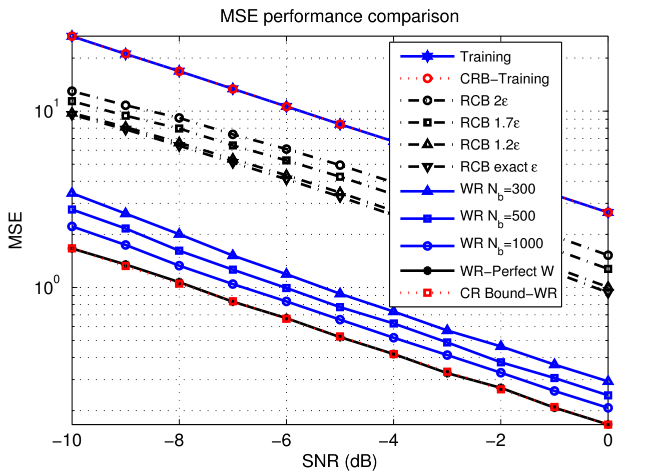

Fig. 1. MSE comparison of SB, robust and training based MIMO MC-CDMA channel estimators.

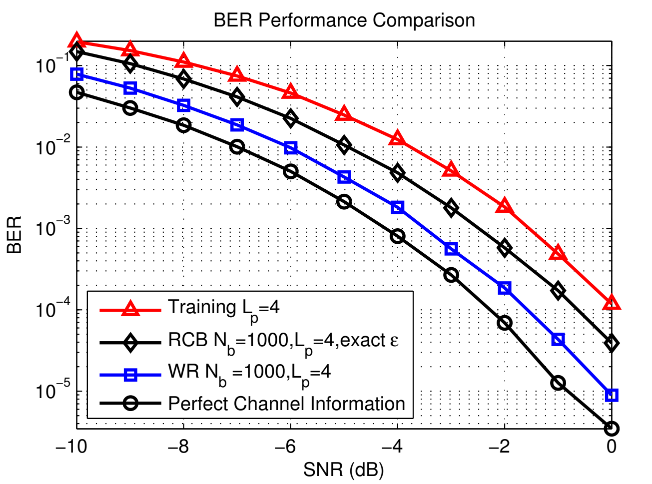

Fig. 2. BER performance comparison of the SB, robust and training based MIMO MC-CDMA channel estimation schemes.

the robust capon beamformer (RCB) [12]. Thus, the robust estimator partially employs the blind statistical information present in the covariance matrix $\mathbf{R}_d$. However, as seen from the results presented in the next section, the MSE performance of the robust estimator, even though slightly superior to that of the training based estimator, is significantly worse compared to that of the WR based SB estimator.

VI. Simulation and Results

We simulated a frequency-selective 4×2 MIMO MC-CDMA downlink, i.e. with $N_r = 4$ receive antennas and $N_t = 2$ transmit antennas, with a delay spread of $L_h = 4$. The system comprised of $K = 12$ active users with spreading sequence length $N = 256$.

We begin by comparing the performance of the competing semi-blind (16), training (18), and robust schemes (20) for MIMO MC-CDMA channel estimation. From the results in Fig.1 for number of blind symbols $N_b = 300, 500, 1000$, with $L_p = 12$ pilot symbols, it can be seen that the semi-blind estimate results in significantly lower MSE in the channel estimate compared to the conventional training based estimator. Further, the MSE of the SB estimator progressively attains the CRB for SB estimation derived in (17). Also, the MSE of the uncertainty set based robust estimator is plotted for different uncertainty parameter values ϵ. Though the performance of the robust estimator is superior to that of the training based estimator, it can be seen to have a substantially poor performance compared to the SB estimator. Also, from the BER performance comparison of the different estimation schemes, with $L_p = 4$ pilot symbols and $N_b = 1000$ symbols, illustrated in Fig.2, it can be seen that the SB estimator results in SNR gains of 2 dB and 1.2 dB relative to the training and robust estimates respectively. Hence, overall, the SB estimator achieves a significantly lower MSE of estimation and significantly reduced BER amongst the competing channel estimation schemes for different scenarios.

VII. Conclusion

A novel whitening-rotation based semi-blind scheme has been proposed in the context of frequency-selective MIMO MC-CDMA channel estimation. The proposed scheme is based on the MMD based receiver structure developed for the MIMO MC-CDMA system and hence is of lower computational complexity compared to existing approaches for frequency-selective MIMO MC-CDMA channel estimation. Further, a lower bound on the MSE of the proposed SB estimator has been derived using the framework for CRB for constrained parameters. The proposed scheme is seen to have a significantly superior performance compared to training and robust estimation approaches.

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