A Synergic Search for QCD Critical Point

Rajiv V. Gavai
T. I. F. R., Mumbai

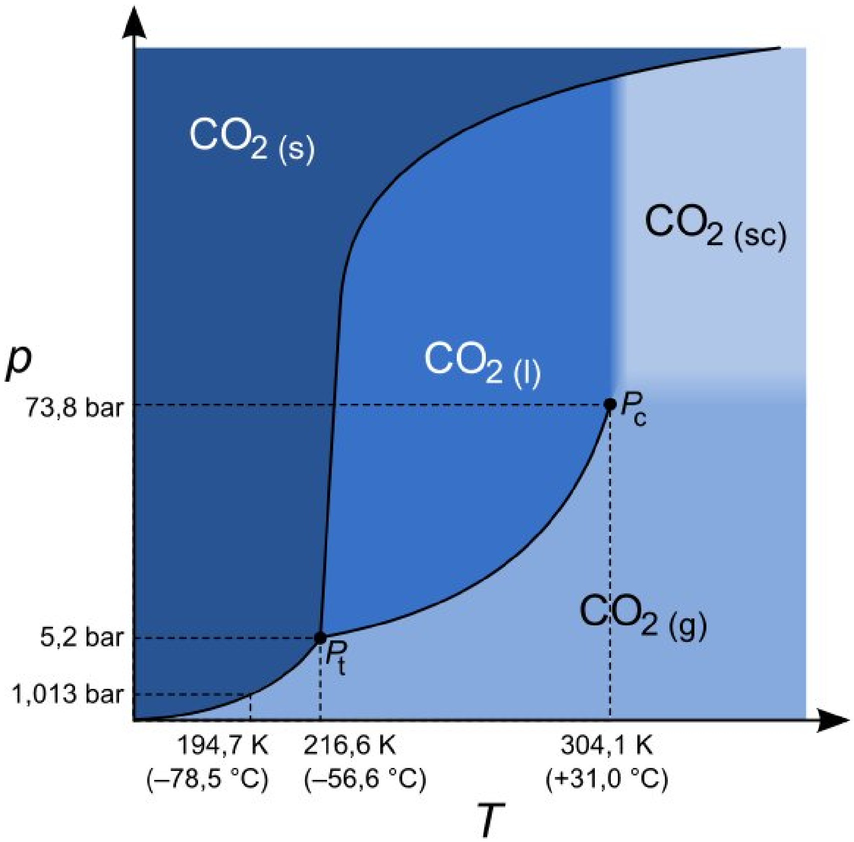
Importance of Being Critical

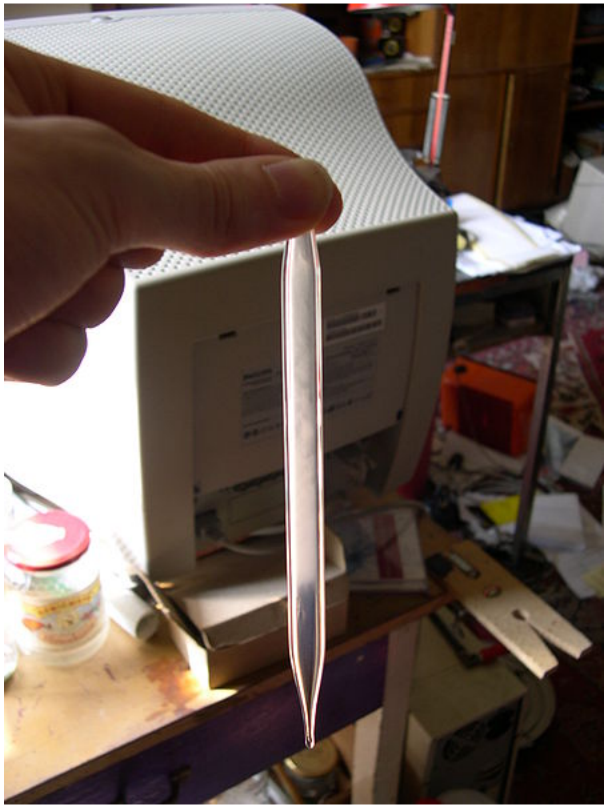
Theoretical Results

Searching Experimentally

Summary

Importance of Being Critical : meV Critical Point





$\hbar = c = k = 1 \Longrightarrow$ 1.16 $\times 10^4\,{}^\circ\mathrm{K} \equiv$ 1 eV; Picts From Wikipedia

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♡ About a third of hop extraction using supercritical CO_2!

The MeV Scale – QCD – Critical Point

- QCD : A Gauge Theory of interactions of quarks-gluons.
- *Unlike QED*, the coupling is usually very large : by $\sim$ 100.
- For (N_f) massless particles, Chiral Symmetry $(SU(N_f) \times SU(N_f))$.
- Much richer structure : Quark Confinement, Chiral Symmetry Breaking..

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- Much richer structure : Quark Confinement, Chiral Symmetry Breaking..
- Very high interaction (binding) energies. E.g., $M_{Proton} \gg (2m_u + m_d)$, by a factor of 100 $\rightarrow$ Understanding it is knowing where the Visible mass of Universe comes from.
- Interactions break the chiral symmetry dynamically, leading to effective masses for the quarks.

- Light pions (m_π =0.14 GeV) and heavy baryons (protons/neutrons; m_N=0.94 GeV) arise this way (Y. Nambu, Physics Nobel Prize 2008).

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- Chiral symmetry **may** get restored at sufficiently high temperatures or densities. Effective mass then 'melts' away, just as magnet loses its magnetic properties on heating.

- New States at High Temperatures/Density expected on basis of models.

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- Quark-Gluon Plasma is such a phase. It presumably filled our Universe a few microseconds after the Big Bang & can be produced in Relativistic Heavy Ion Collisions. QCD Critical Point arises also due to Chiral Symmetry.

- Ideally, QCD should shed light on its richer structure : Quark Confinement, Dynamical Symmetry Breaking.. But Models did that first.

QCD Phase diagram

♠ A fundamental aspect – Critical Point in T-μ_B plane;

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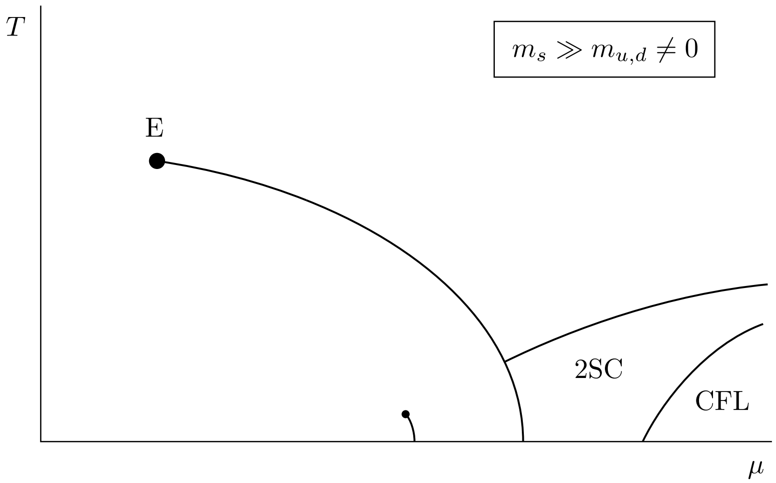

From Rajagopal-Wilczek Review

QCD Phase diagram

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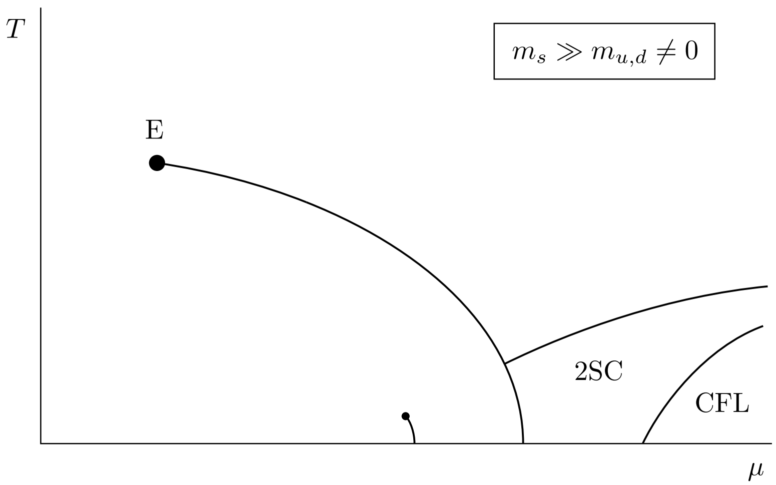

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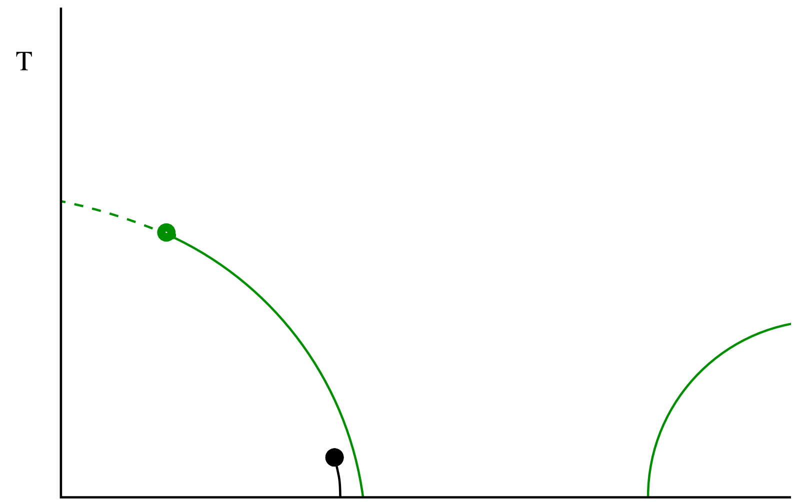

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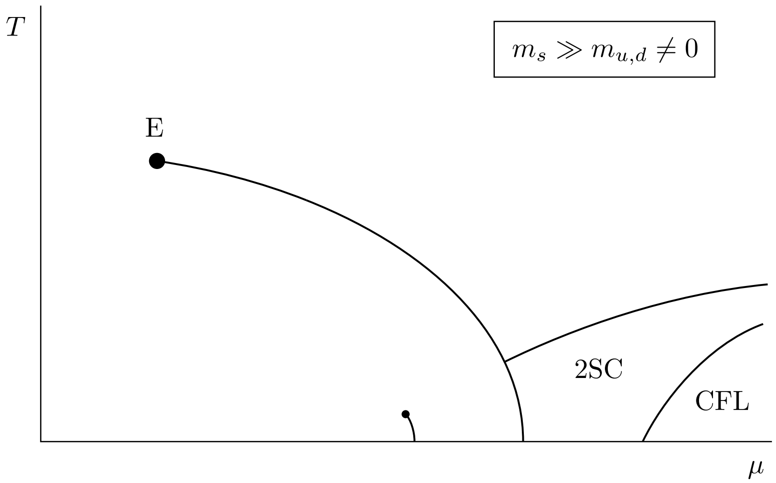

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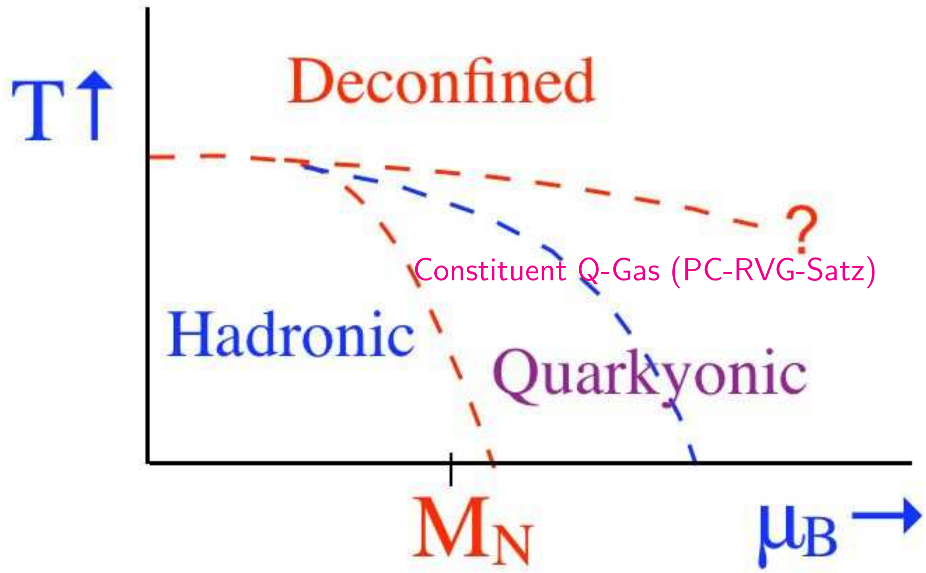

Putting QCD to Work

- QCD Partition Function : $Z_{QCD} = \mathrm{Tr}\ \exp[-(H_{QCD} - \mu_B N_B)/T]$.

- A first-principles calculation of $\epsilon(\mu, T)$ or $P(\mu, T)$ to look for phase transitions, Critical Point and many phases using the underlying theory QCD alone: NO free parameters and NO arbitrary assumptions.

- Price to pay : Functional integrations have to be done over quark and gluon fields : $\int\ dx\ F(x) \rightarrow \int\ \mathcal{D}\phi\ \mathcal{F}[\phi(x)]$.

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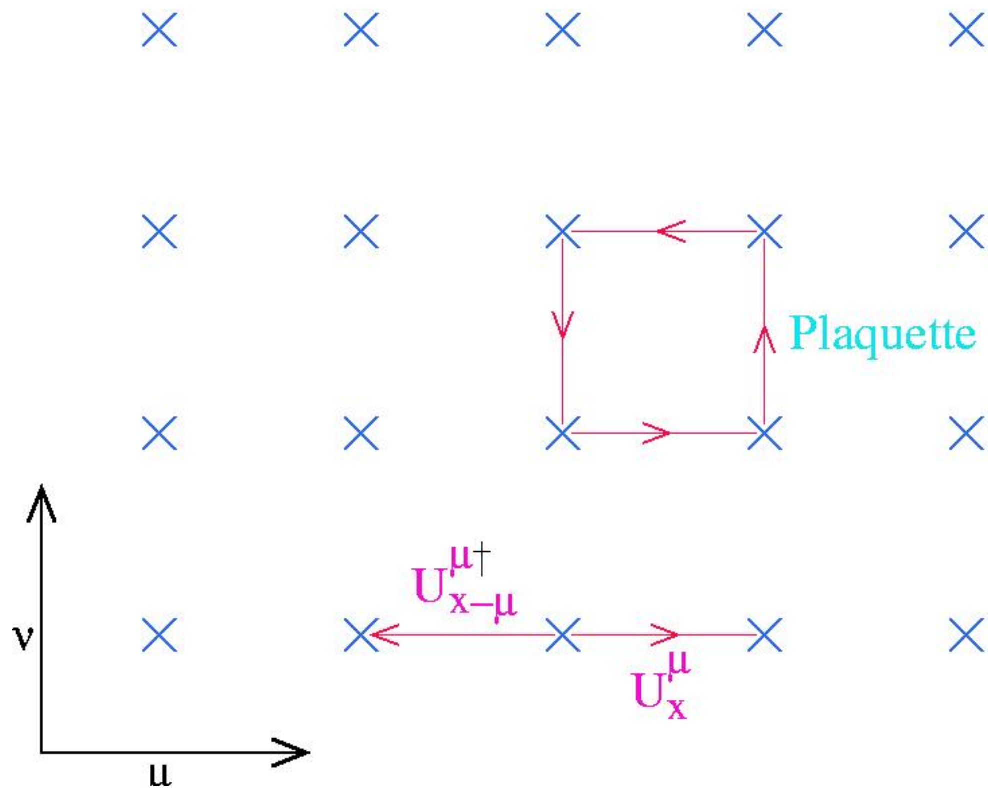
- Simpson integration trick : $\int\ dx\ F(x) = \lim_{\Delta x \to 0} \sum_i\ \Delta x\ F(x_i)$.

- Its analogue to perform functional integrations needs discretizing the space-time on which the fields are defined : Lattice Field Theory !

Basic Lattice QCD

- Discrete space-time : Lattice spacing a UV Cut-off.
- Quark fields $\psi(x)$, $\bar{\psi}(x)$ on lattice sites.
- Gluon Fields on links : $U_\mu(x)$



Basic Lattice QCD

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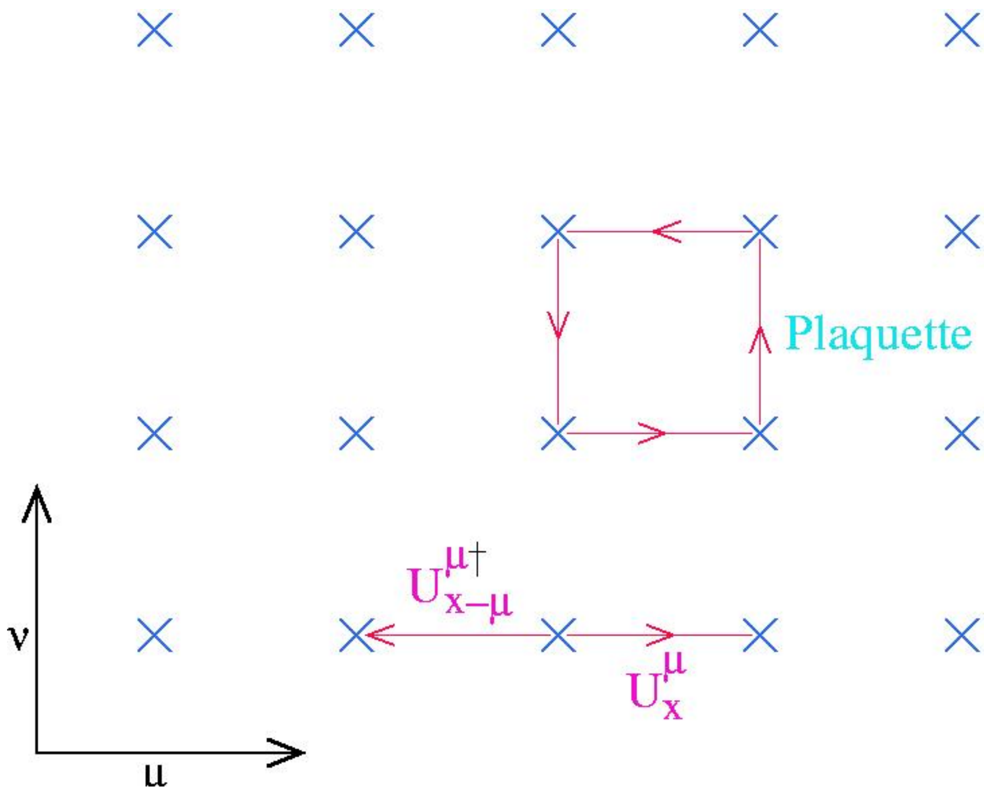
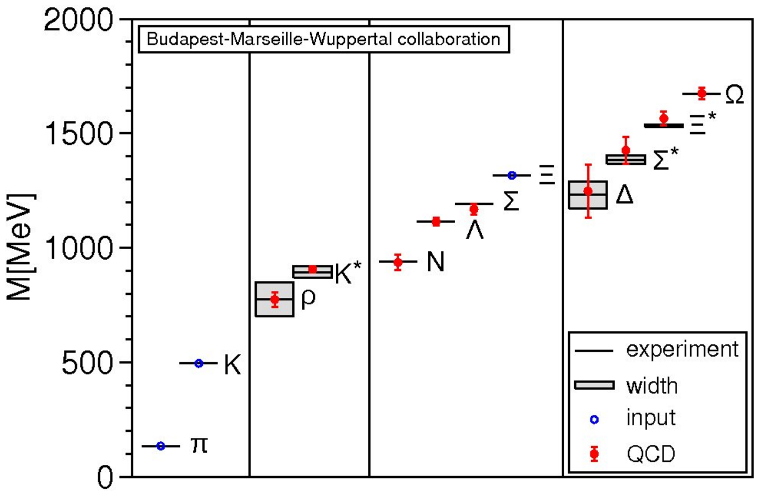
- Quark fields $\psi(x)$, $\bar{\psi}(x)$ on lattice sites.
- Gluon Fields on links : $U_\mu(x)$
- Gauge invariance : Actions from Closed Wilson loops, e.g., plaquette.
- Fermion Actions : Staggered, Wilson, Overlap, Domain Wall..



Lattice QCD Results

- QCD defined on a space time lattice – Best and Most Reliable way to extract non-perturbative physics: Notable successes are hadron masses(S. Dürr et all, Science (2008)) & decay constants.



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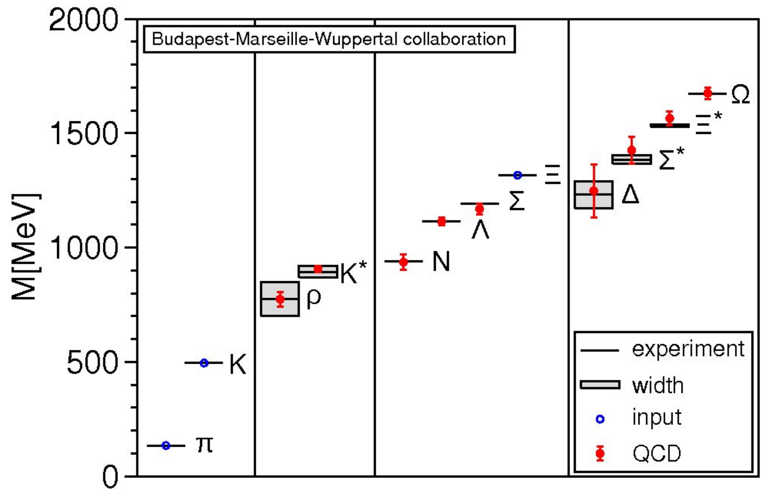
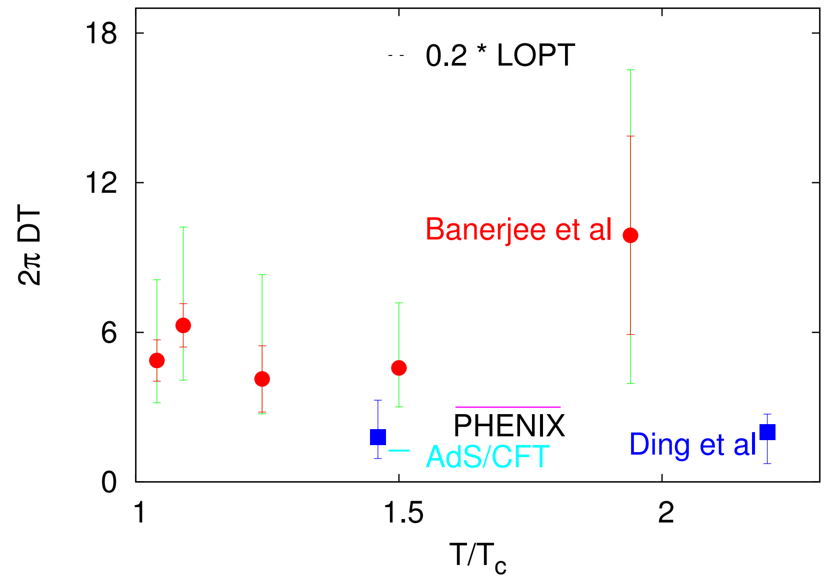
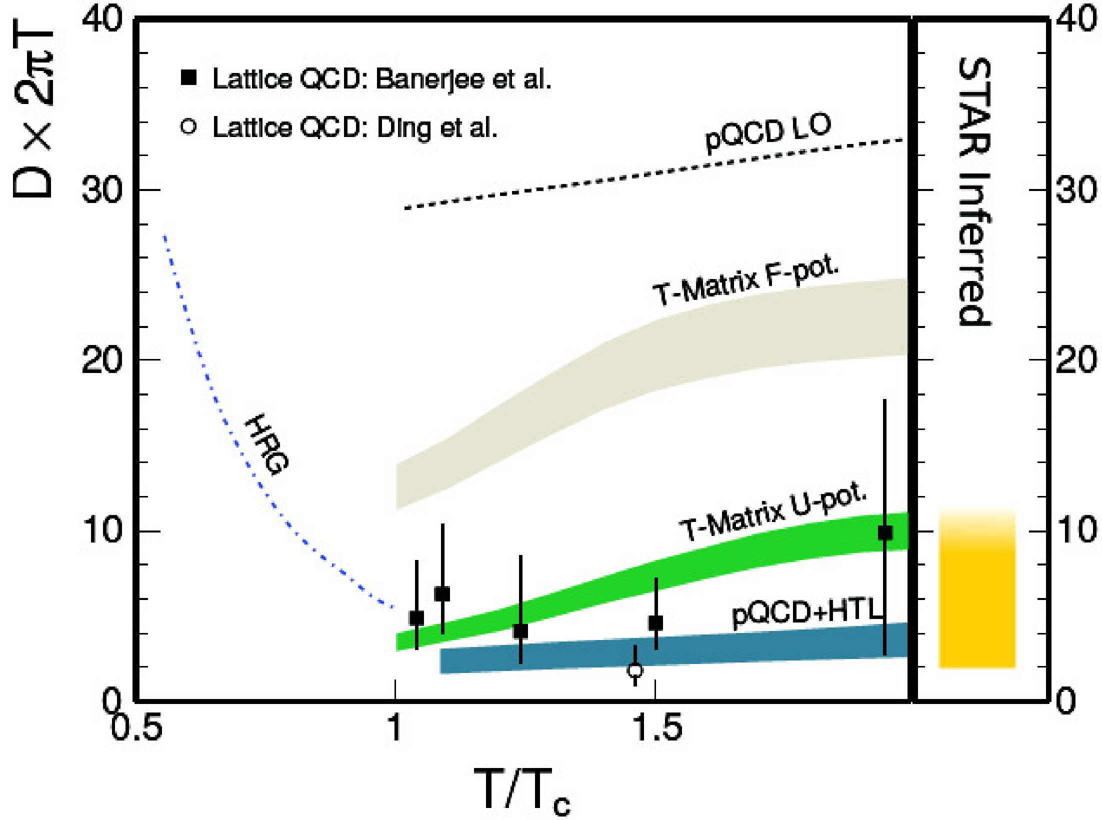




- The Transition Temperature T_c, the Equation of State, Heavy flavour diffusion coefficient D (Banerjee et al. PRD (2012)), Flavour Correlations C_{BS} and the Wróblewski Parameter λ_s are some examples for Heavy Ion Physics.

Diffusion coefficient



- Compatible with models predicting a value of diff. coefficient between 2 to ~10
- Lattice calculations, although with large uncertainties, are consistent with values inferred from data

Obstacles for $\mu \neq 0$

- Quark type : For $\langle\bar{\psi}\psi\rangle$ to remain Order Parameter, Chiral Symmetry on Lattice Crucial $\rightsquigarrow$ Staggered fermions.

- Only two light flavours results in a Critical Point. $U_A(1)$-anomaly may be important as well. Staggered fermions break flavour symmetry and $U_A(1)$!

- Overlap/Domain Wall quarks required. Nonzero μ difficult problem for them but resolved recently. (RVG-Sharma PLB '15, PRD '12, PLB '12, PRD '10; Bloch-Wittig PRL '06, PRD '07; Banerjee-RVG-Sharma PRD '08,PoS LAT '08).

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- Complex Measure : Probabilistic methods used to compute, with a measure $\sim \exp(-S_G)$ Det M. Simulations can be done IF Det $M > 0$ for all sets of gauge fields. However, Det M is a complex number for any $\mu \neq 0 \rightsquigarrow$ The Phase/sign problem.

Lattice Approaches

Several Approaches proposed in the past two decades : None as satisfactory as the usual $T \neq 0$ simulations. Still scope for a good/great idea !

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  - Two parameter Re-weighting (Z. Fodor & S. Katz, JHEP 0203 (2002) 014).
  - Imaginary Chemical Potential (Ph. de Frocrand & O. Philipsen, NP B642 (2002) 290; M.-P. Lombardo & M. D'Elia PR D67 (2003) 014505).
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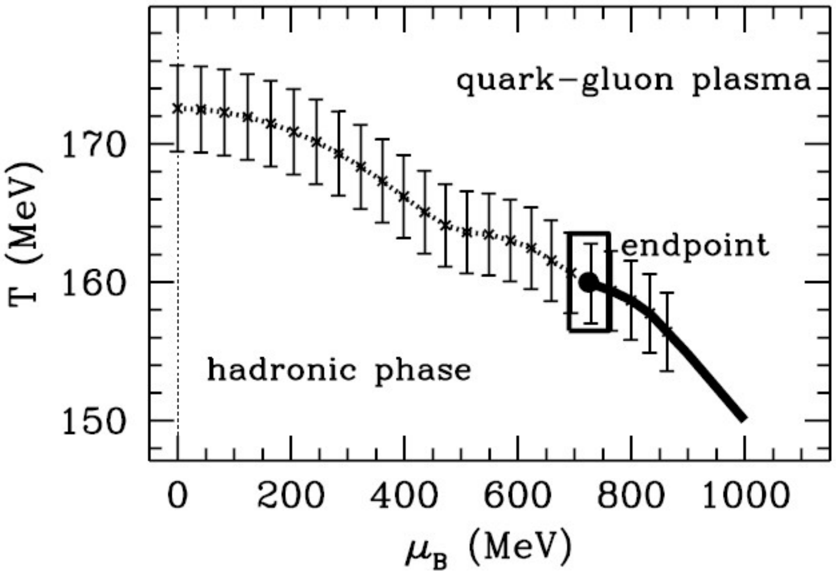
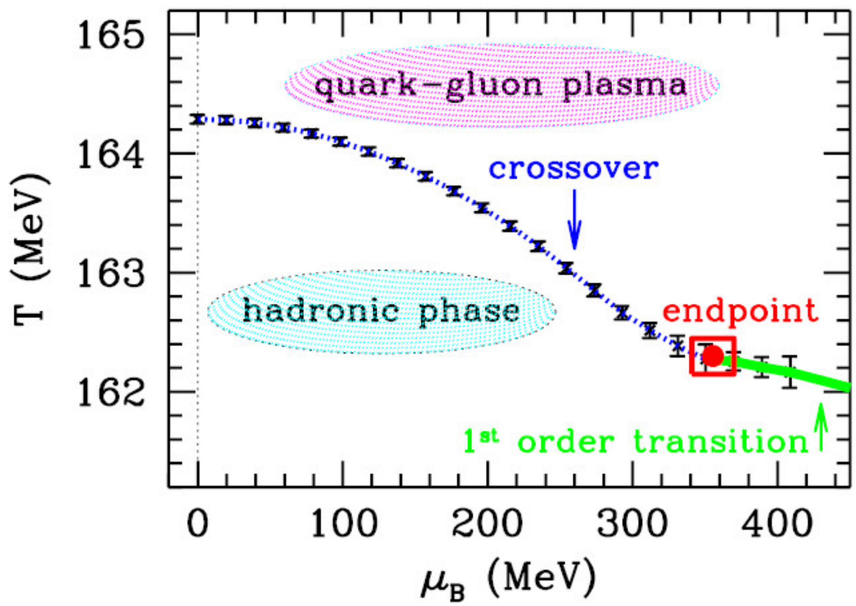
- Why Taylor series expansion? — i) Ease of taking continuum and thermodynamic limit & ii) Better control of systematic errors.

First Glimpse of QCD Critical Point





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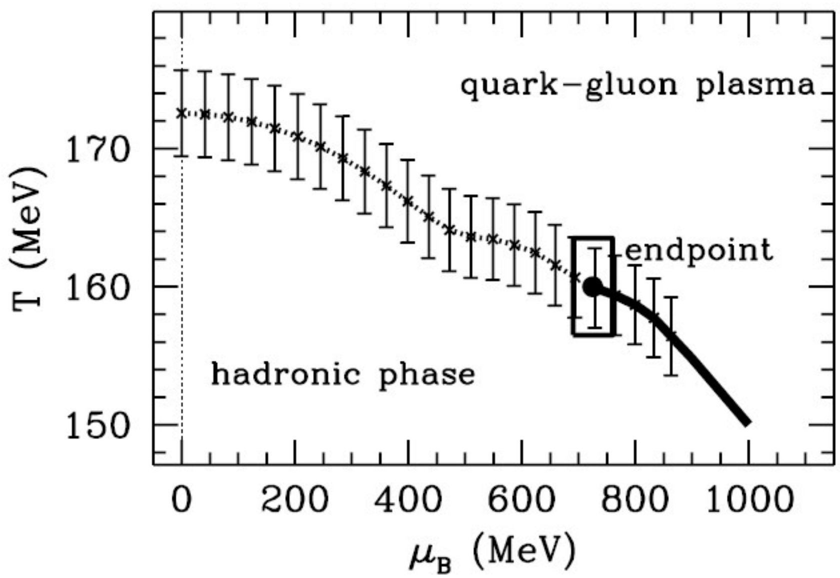

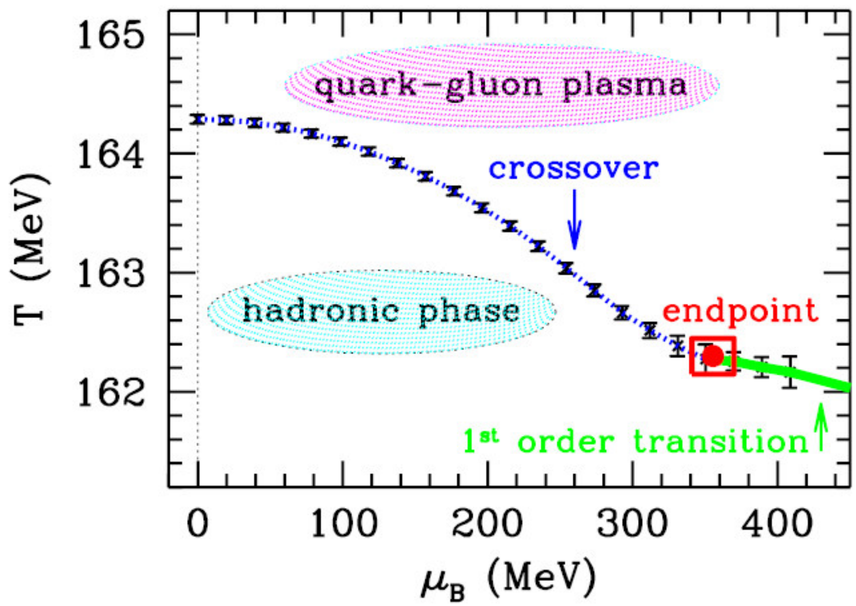

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Larger N_t or Continuum limit ?

QCD Critical Point : Taylor Expansion

- $1^{\rm st}$ & $2^{\rm nd}$ derivatives with μ_i yield various number densities and susceptibilities. Denoting higher order susceptibilities by χ_{n_u,n_d}, the pressure has the expansion:

$$\frac{\Delta P}{T^4} \equiv \frac{P(\mu,T)}{T^4} - \frac{P(0,T)}{T^4} = \sum_{n_u,n_d} \chi_{n_u,n_d}(\mu_i/T=0)\,\frac{1}{n_u!}\left(\frac{\mu_u}{T}\right)^{n_u}\frac{1}{n_d!}\left(\frac{\mu_d}{T}\right)^{n_d}.$$

- Using this, a series for baryonic susceptibility can be constructed. Its radius of convergence, obtained by cannonical methods, is the nearest critical point.

- All coefficients of the series must be POSITIVE for the critical point to be at real μ, and thus physical.

- We (ILGTI-Mumbai '05, '09, '13) use up to 8^{th} order. Budapest-Wuppertal & Bielefeld-RBC so far have up to 6^{th} order. Ideas to extend to higher orders are emerging (Gavai-Sharma PRD 2012 & PRD 2010).

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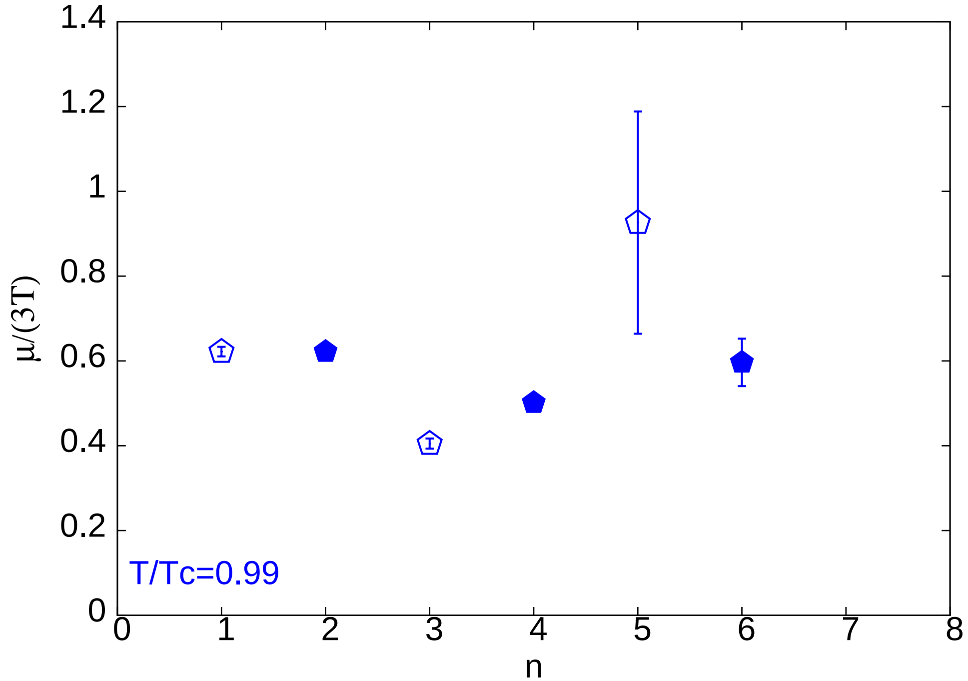
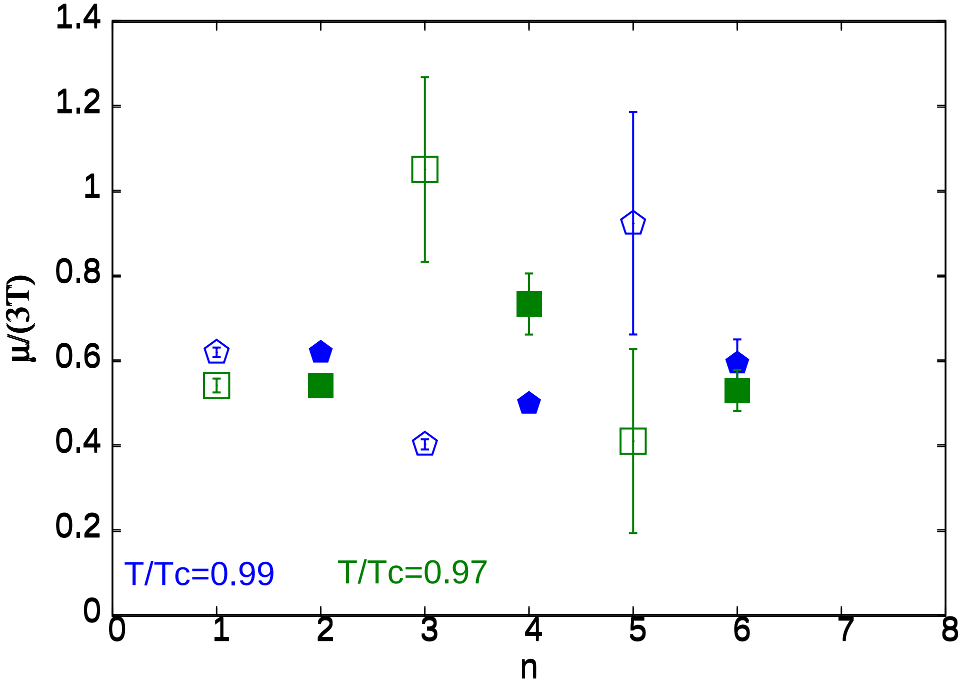
Simulation Details & Results

- Staggered fermions with $N_f = 2$ of $m/T_c = 0.1$; R-algorithm used.
- Continuum limit of $a \to 0$ by holding $T_c^{-1} = aN_t = constant$ as $N_t \uparrow$.
- T_c — defined by the peak of a susceptibility (of Polyakov loop) at $\mu = 0$.
- Began with Lattices : 4 $\times N_s^3$, $N_s =$ 8, 10, 12, 16, 24 (Gavai-Gupta, PRD 2005);
  Finer Lattice : 6 $\times N_s^3$, $N_s =$ 12, 18, 24 (Gavai-Gupta, PRD 2009).
- Even finer Lattice : 8 $\times 32^3$ (Datta-RVG-Gupta, NPA 2013).
  Aspect ratio, N_s/N_t, maintained four to reduce finite volume effects.
- Simulations made at $T/T_c =$ 0.90, 0.92, 0.94, 0.96, 0.98, 1.00, 1.02, 1.12, 1.5 and 2.01.


1.4
1.2
1
0.8
0.6
0.4
0.2
0
μ/(3T)
0
1
2
3
4
5
6
7
8
n
T/Tc=0.99


1.4
1.2
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0.8
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μ/(3T)
T/Tc=0.99
T/Tc=0.97
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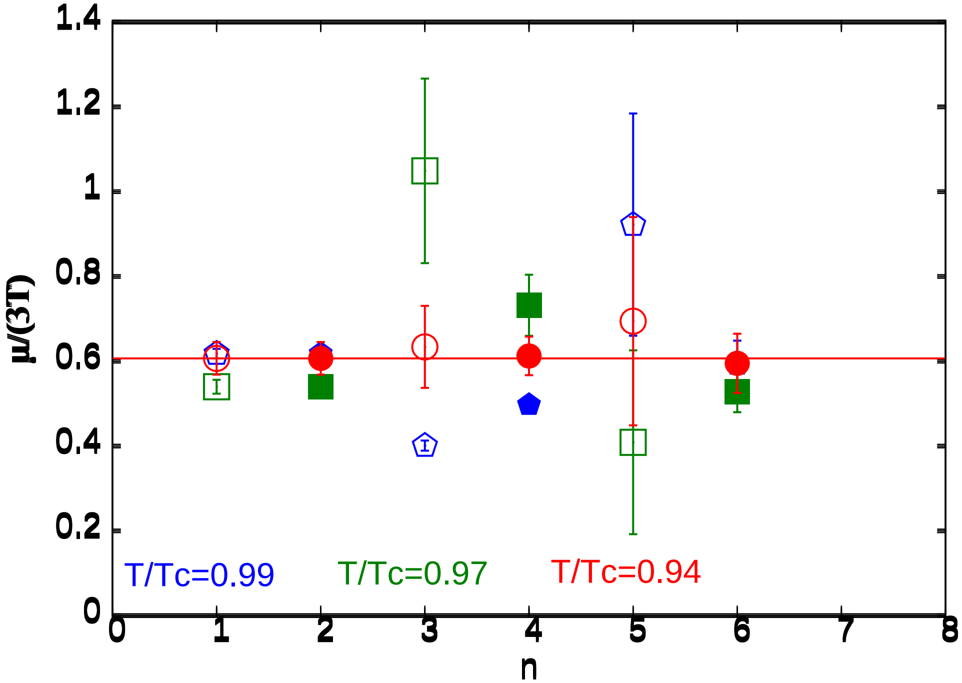

1.4
1.2
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T/Tc=0.97
T/Tc=0.94
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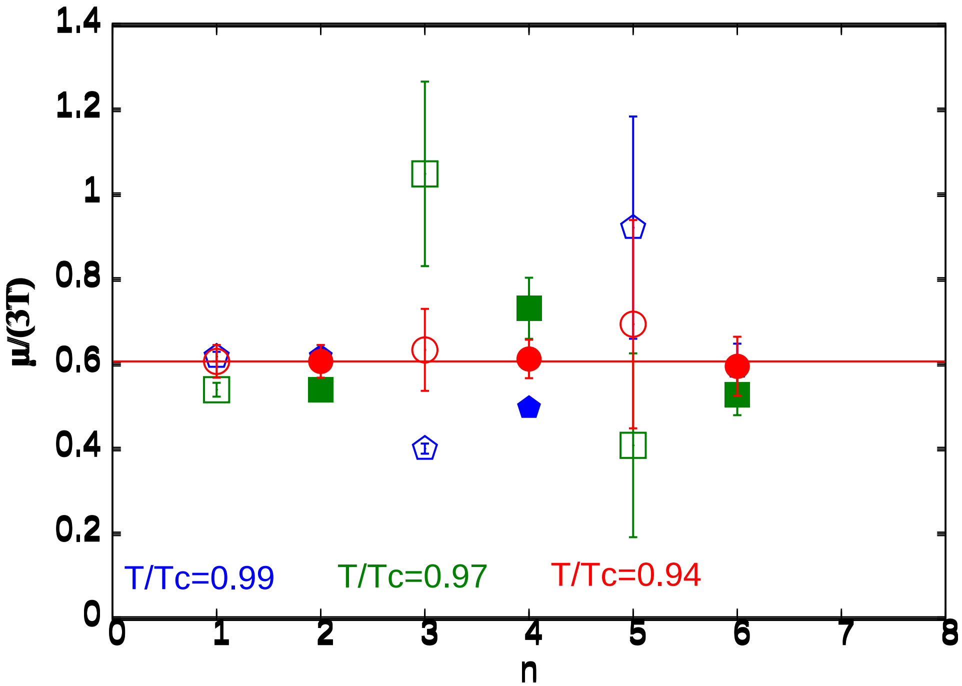
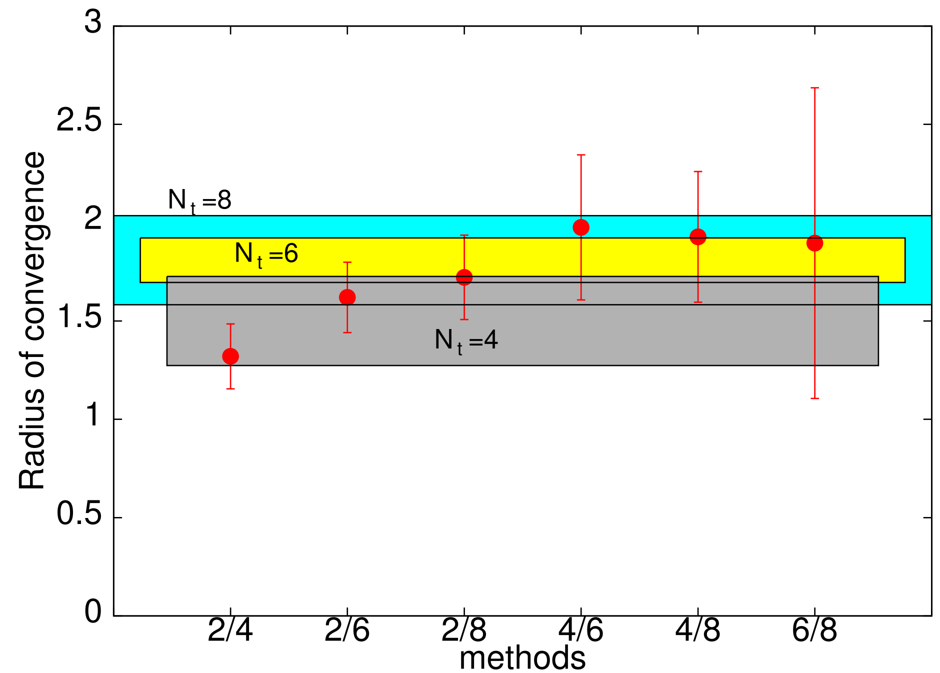
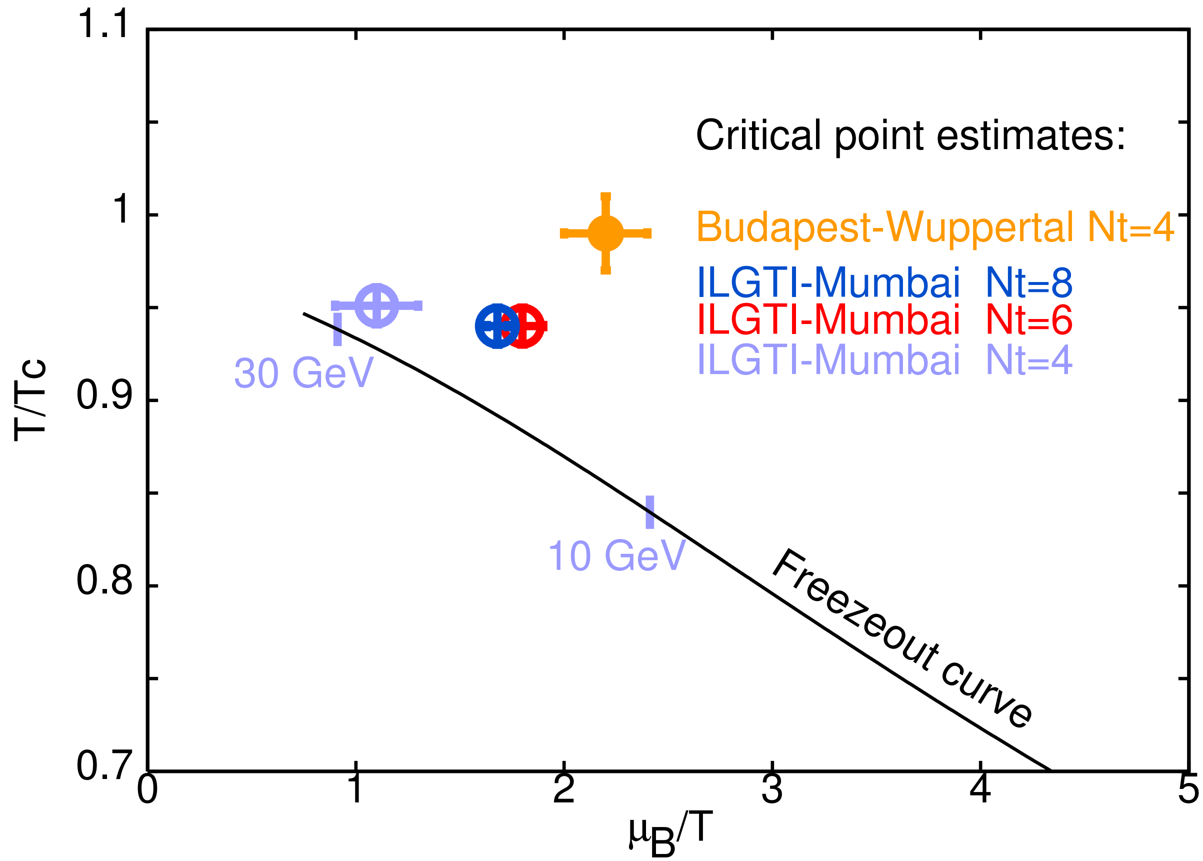




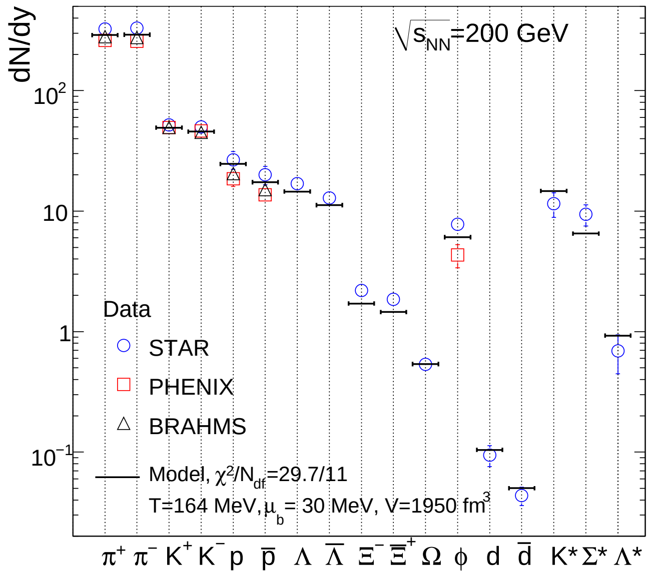
- $\frac{T^E}{T_c} = 0.94 \pm 0.01$, and $\frac{\mu_B^E}{T^E} = 1.8 \pm 0.2 (1.8 \pm 0.1)$ for the $N_t = 8(6)$ lattice (Datta-RVG-Gupta, '08, '13). Recent high statistics coarser $(N_t = 4)$ lattice result was $\mu_B^E / T^E = 1.5 \pm 0.2$ (Gupta-Kartik-Majumdar PRD '14).
- Critical point at $\mu_B/T \sim 1 - 2$ suggested.

Critical Point : Inching Towards Continuum



Searching Experimentally: The freezeout curve

- Hadron yields well described using Statistical Hadronization Models, leading to the freezeout curve in the T-μ_B plane. (Andronic, Braun-Munzinger & Stachel, PLB 2009 ; Oeschler, Cleymans, Redlich & Wheaton, 2009)



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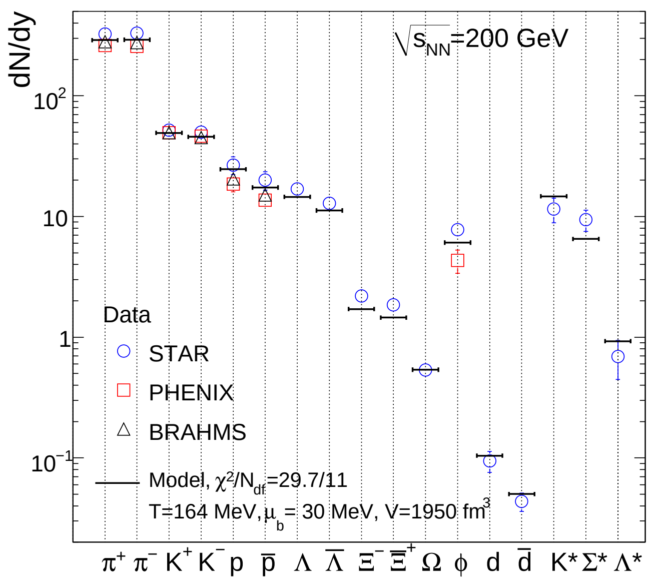
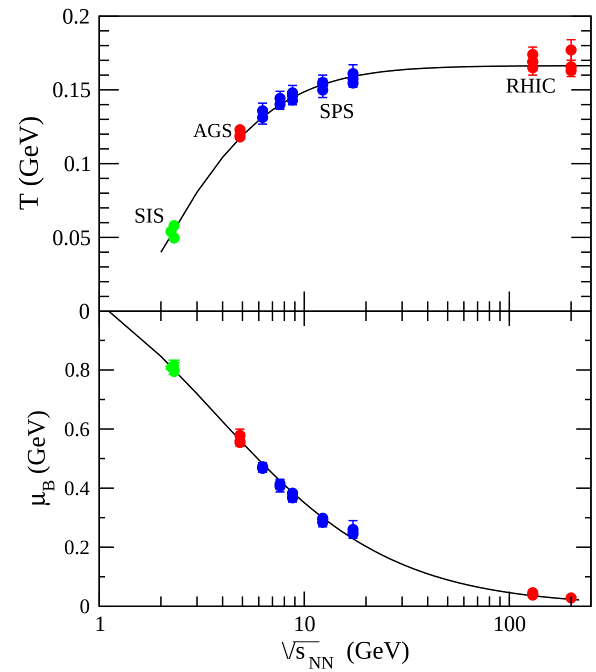




- Plotting these results in the T-μ_B plane, one has the freezeout curve, which was shown to correspond the $\langle E \rangle / \langle N \rangle \simeq 1$. (Cleymans and Redlich, PRL 1998)

The Beam Energy Scan Program at RHIC and SPS

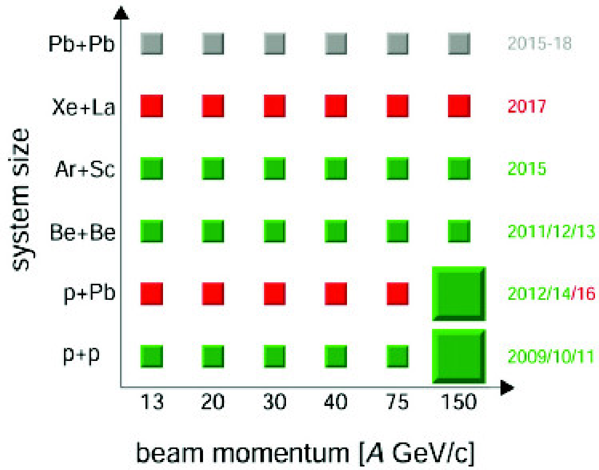
RHIC: STAR and PHENIX (Collider)

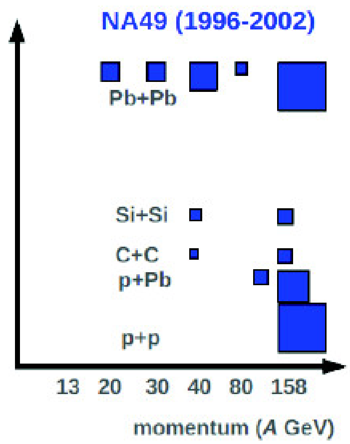
Au+Au Collisions

√s (GeV)	Statistics(10^6)	μ_B (MeV)
7.7	~4	420
11.5	~12	315
14.5	~ 20	266
19.6	~36	205
27	~70	155
39	~130	115
62.4	~67	70
200	~350	20

arXiv:1007.2613
https://drupal.star.bnl.gov/STAR/starnotes/public/sn0493
https://drupal.star.bnl.gov/STAR/starnotes/public/sn0598

SPS: NA61 and NA49 (Fixed Target)





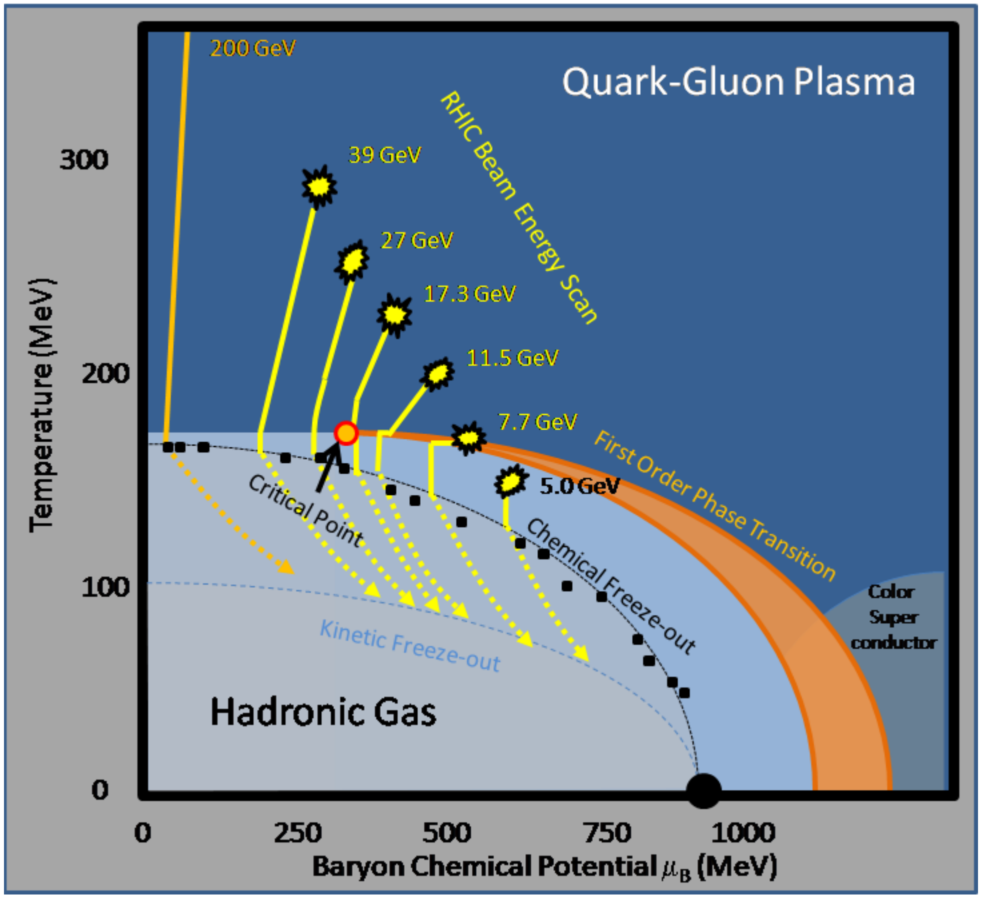
$\sqrt{s_{NN}}$ = 5-17 GeV

JINST 9 (2014) P06005 [arXiv:1401.4699]

Finish Ar+Sc collisions in 2015

Exploring the QCD phase structure by varying the collision energy and/ or system size to change temperature and baryon chemical potential.

Searching Along The Freezeout Curve



STAR Collaboration, Aggarwal et al.
arXiv : 1007.2637

– Exploit the facts i) susceptibilities diverge near the critical point and ii) decreasing $\sqrt{s}$ increases μ_B (Rajagopal, Shuryak & Stephanov PRD 1999).

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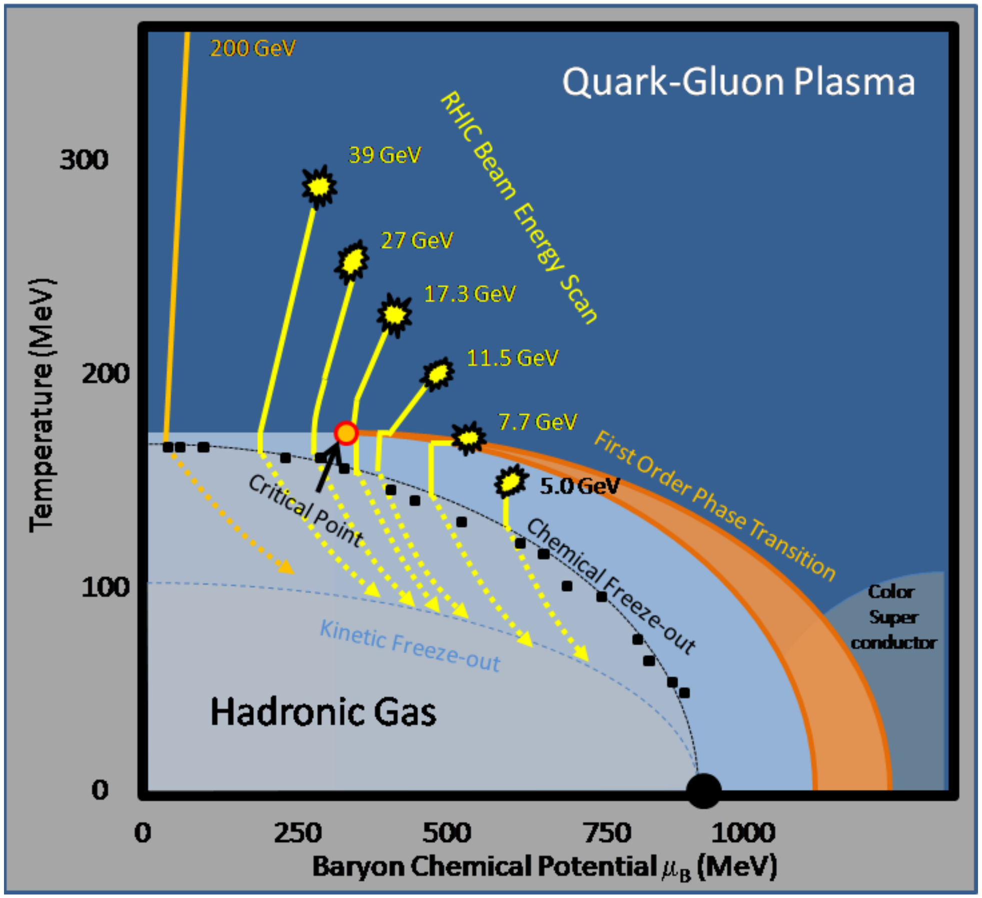

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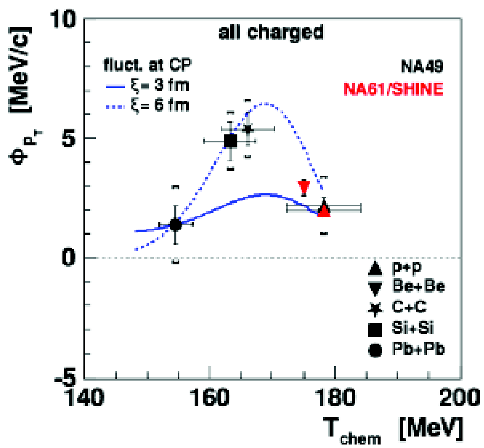
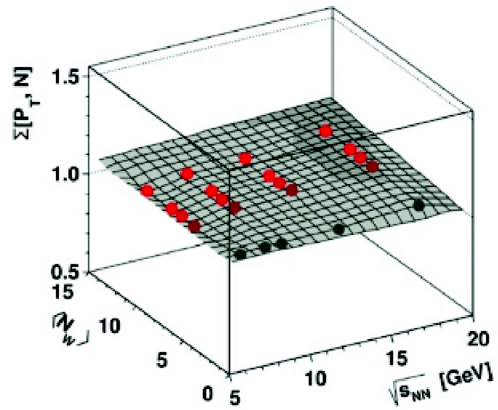
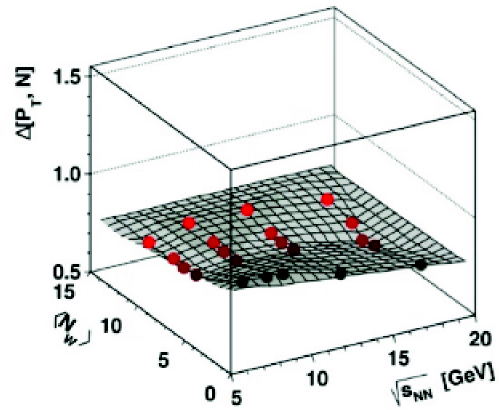
– Look for nonmontonic dependence of the event-by-event fluctuations with colliding energy. No indications in early such results for π, K-mesons. E.g., CERN NA49 results (C. Roland NA49, J.Phys. G30 (2004) S1381-S1384).

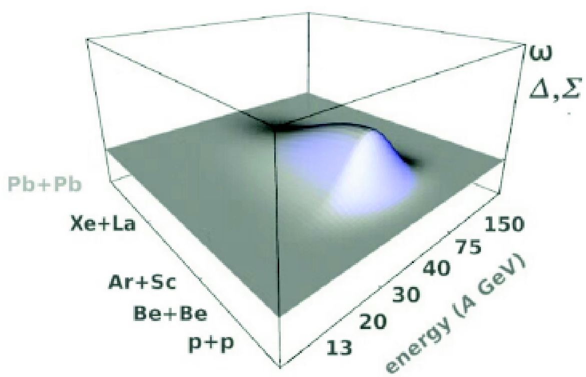
Fluctuations measure from NA49/NA61: 2D Scan

SPS: Scan Nuclear Mass and Collision Energy (2D Scan)









Strongly intensive measure: p+p and Be+Be

$$\Delta^{P_T,N} = \frac{1}{C_\Delta}\left[\langle P_T\rangle \omega(N) - \langle N\rangle \omega(P_T)\right]$$

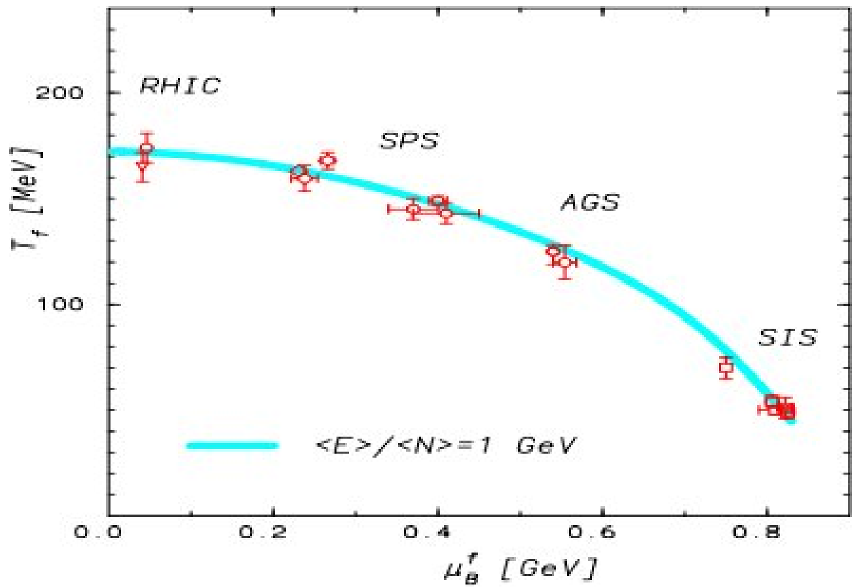
$$\Sigma^{P_T,N} = \frac{1}{C_\Sigma}\left[\langle P_T\rangle \omega(N) + \langle N\rangle \omega(P_T) - 2\left(\langle N\cdot P_T\rangle - \langle N\rangle\langle P_T\rangle\right)\right]$$

Gorenstein, Gazdzicki, Phys.Rev. C84 (2011) 014904
Gazdzicki, et al., Phys.Rev. C88 (2013) 024907

Maja Ma ckowiak-Paw lowska, Mon.,17:00 pm

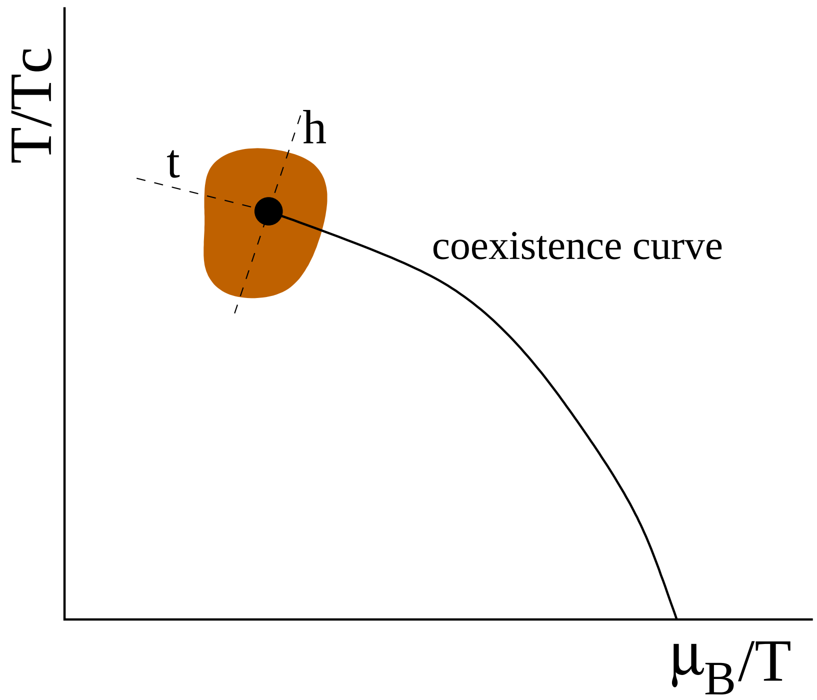
No clear evidence of CP signal.

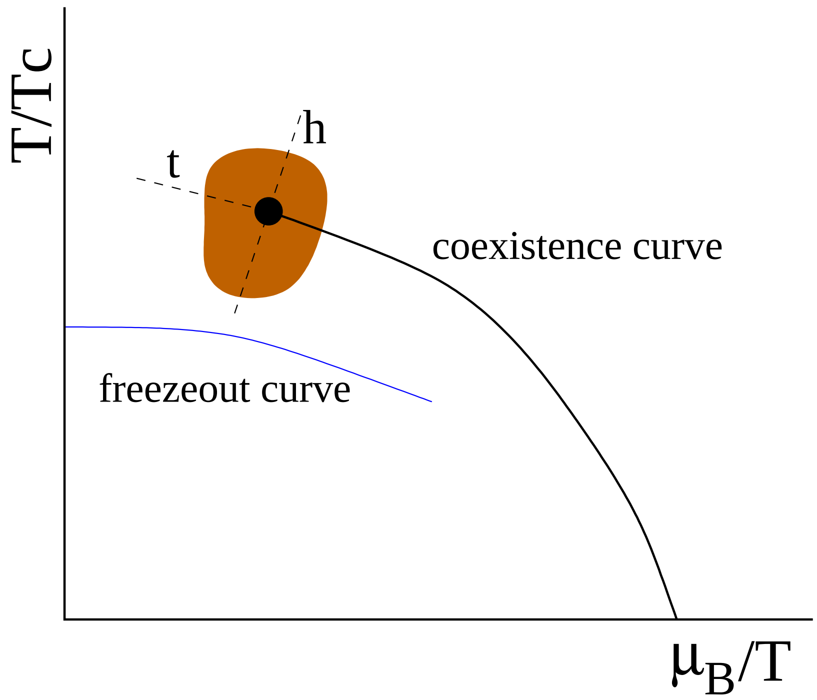
Lattice predictions along the freezeout curve

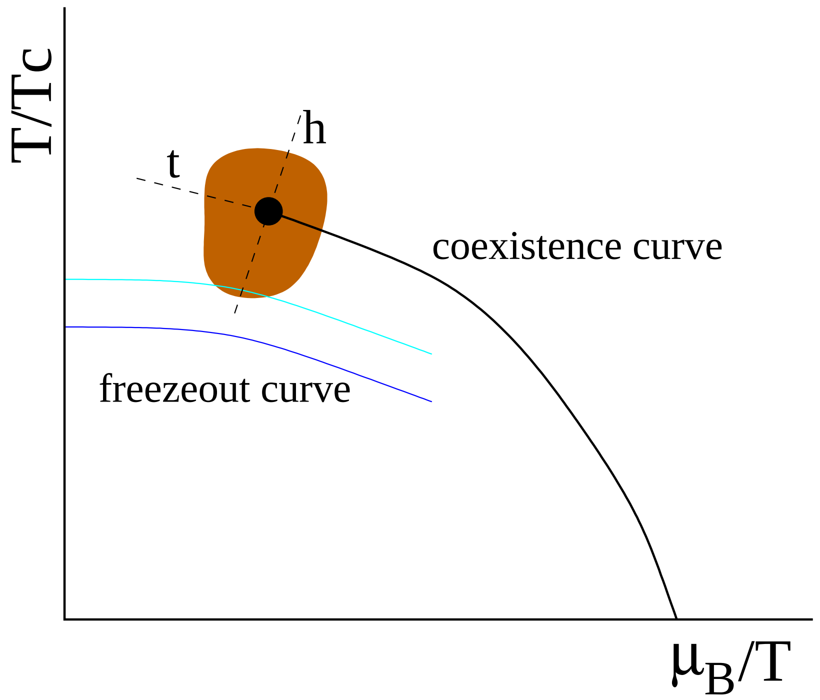


(From Braun-Munzinger, Redlich and Stachel nucl-th/0304013)

- Note : Freeze-out curve is based solely on data on hadron yields, & gives the (T, μ) accessible in heavy-ion experiments.
- Our Key Proposal : Use the freezeout curve from hadron abundances to $predict\ baryon$ fluctuations using lattice QCD along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)


T/Tc
h
t
coexistence curve
μ_B/T


T/Tc
h
t
coexistence curve
freezeout curve
μ_B/T


T/Tc
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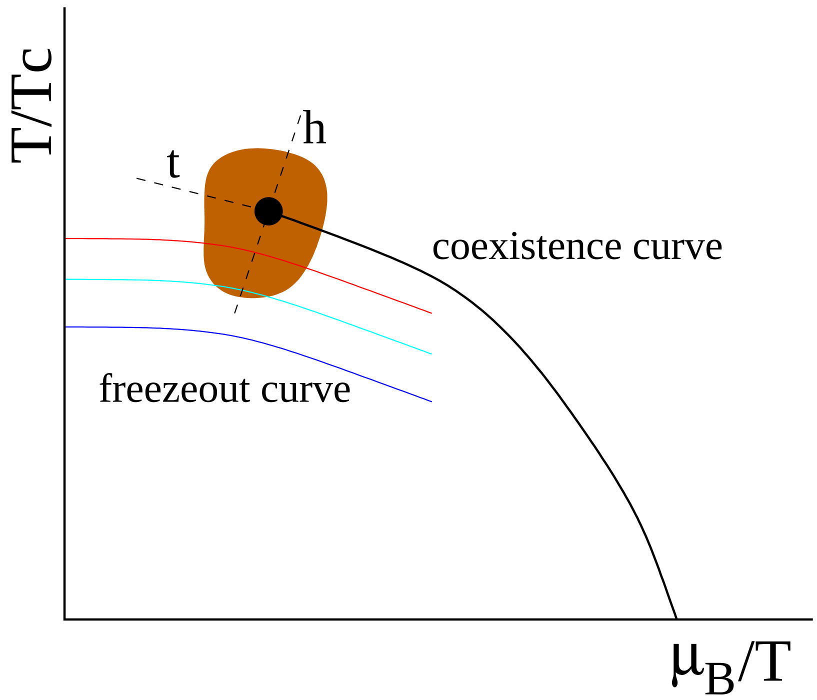
T/Tc
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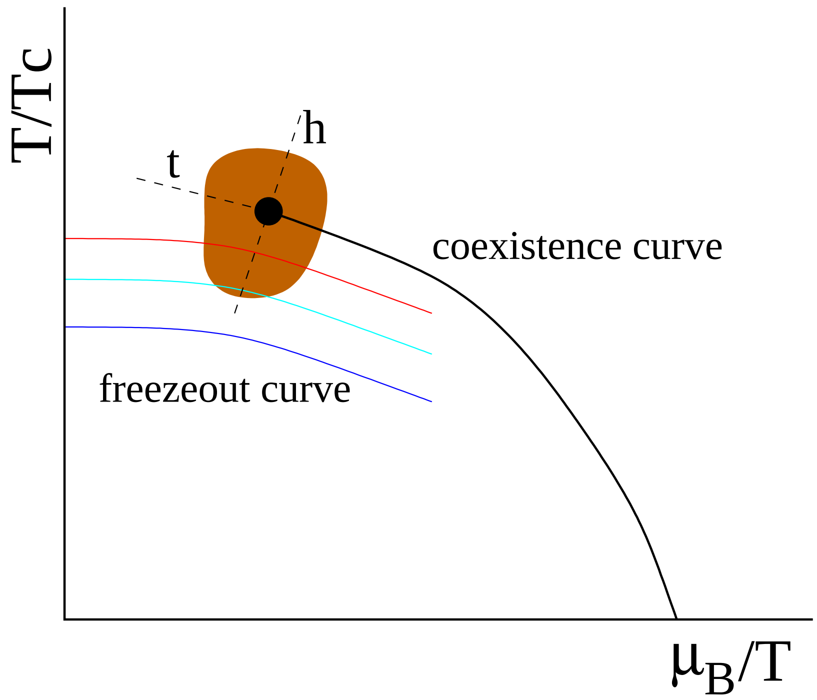
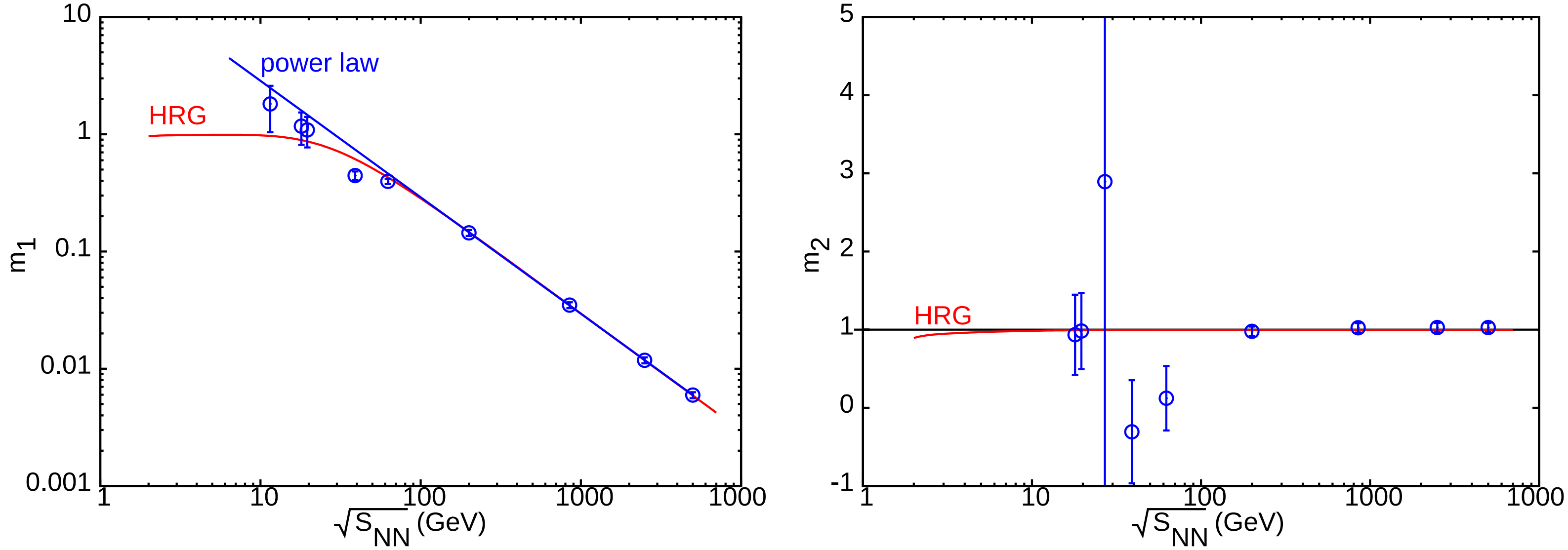
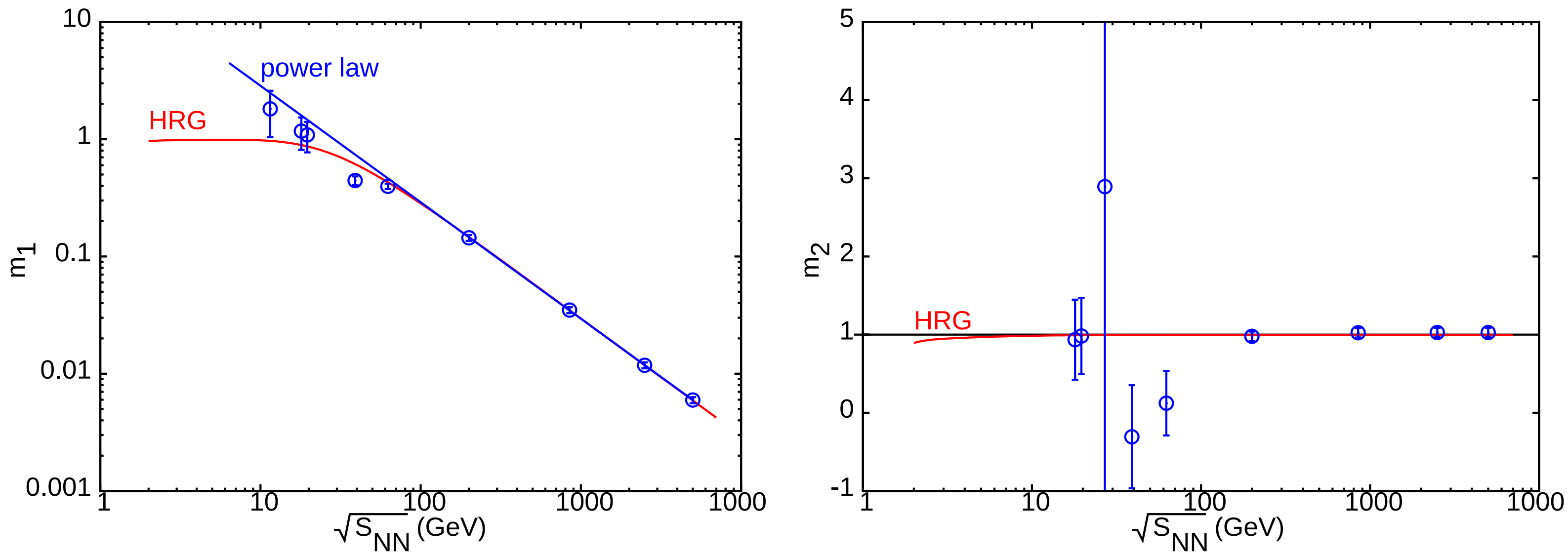


- Use the freezeout curve to relate $(T, \mu_B)$to $\sqrt{s}$ and employ lattice QCD predictions along it. (Gavai-Gupta, TIFR/TH/10-01, arXiv 1001.3796)

- Define $m_1 = \frac{T\chi^{(3)}(T,\mu_B)}{\chi^{(2)}(T,\mu_B)}$, $m_3 = \frac{T\chi^{(4)}(T,\mu_B)}{\chi^{(3)}(T,\mu_B)}$, and $m_2 = m_1 m_3$ and use the Padè method to construct them.



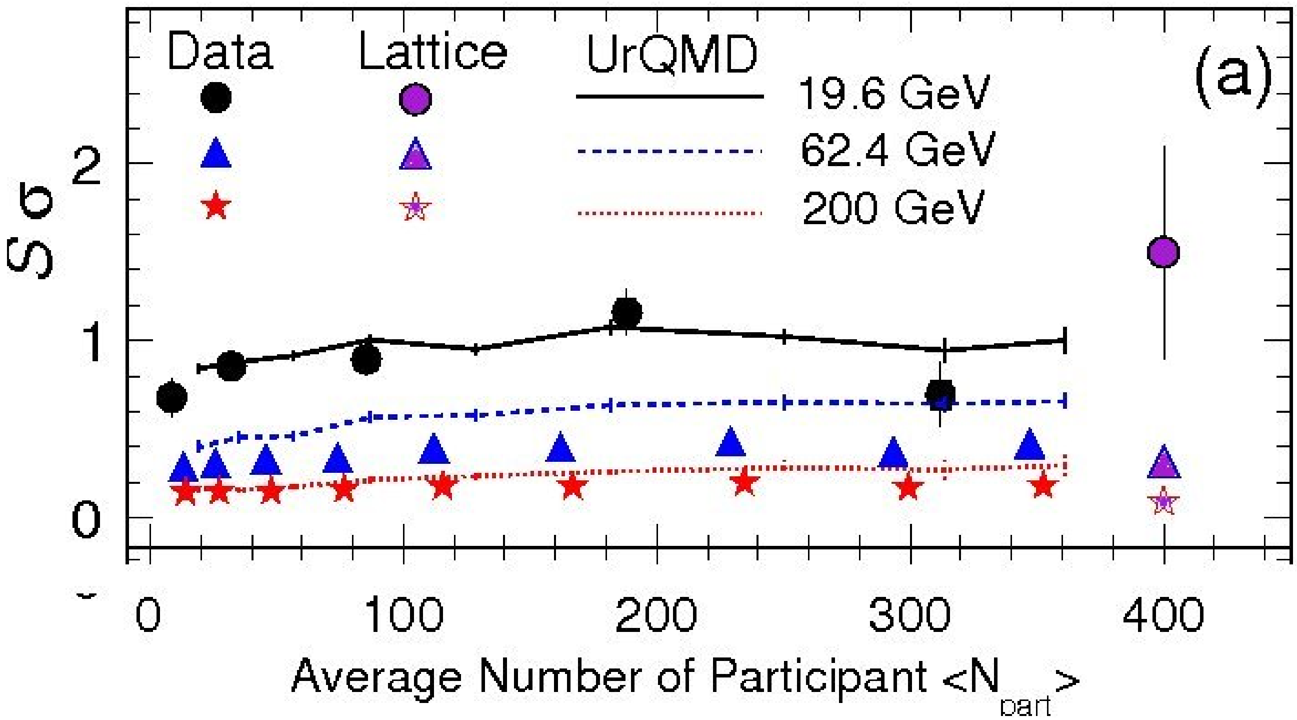
♠ Used $T_c(\mu = 0) = 170$ MeV (Gavai & Gupta, arXiv: 1001.3796).



♠ Used $T_c(\mu = 0) = 170$ MeV (Gavai & Gupta, arXiv: 1001.3796).

- Smooth & monotonic behaviour for large $\sqrt{s}$: $m_1 \downarrow$, $m_3 \uparrow$, and $m_2 \sim$ constant.
- Note that even in this smooth region, an experimental comparison is exciting : Direct Non-Perturbative test of QCD in hot and dense environment.

$S\sigma \equiv m_1$



Aggarwal et al., STAR Collaboration, arXiv : 1004.4959

- Reasonable agreement with our lattice results. Where is the critical point ?

- Our estimated critical point suggests non-monotonic behaviour in all m_i, which should be accessible to the low energy scan of RHIC BNL !

- Caution : Experiments measure *only* proton number fluctuations.

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- In the vicinity of a critical point Proton number fluctuations may suffice.(Hatta-Stephenov, PRL 2003)

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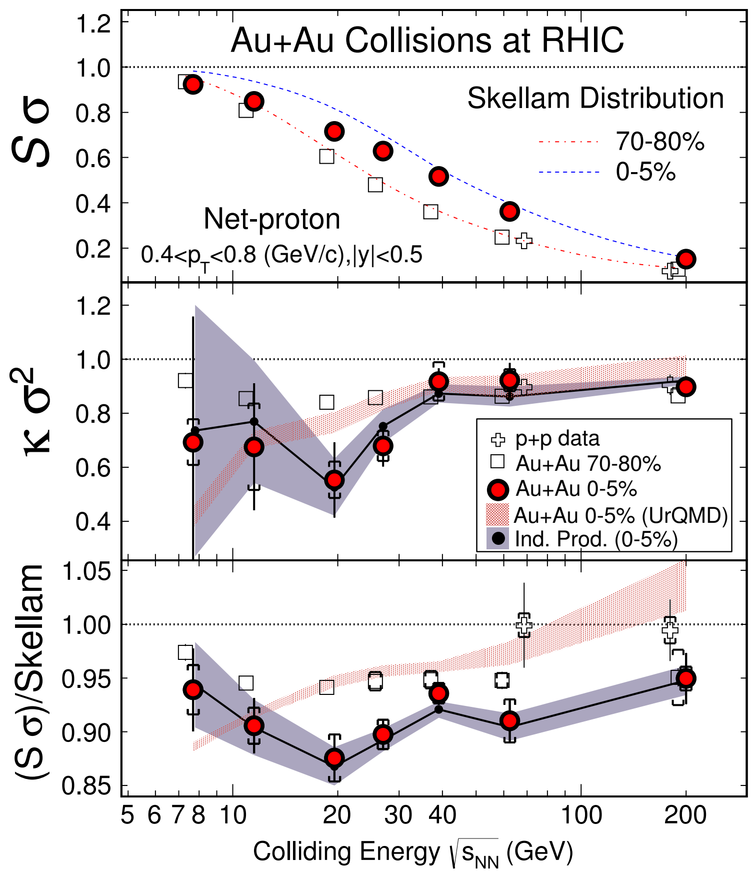
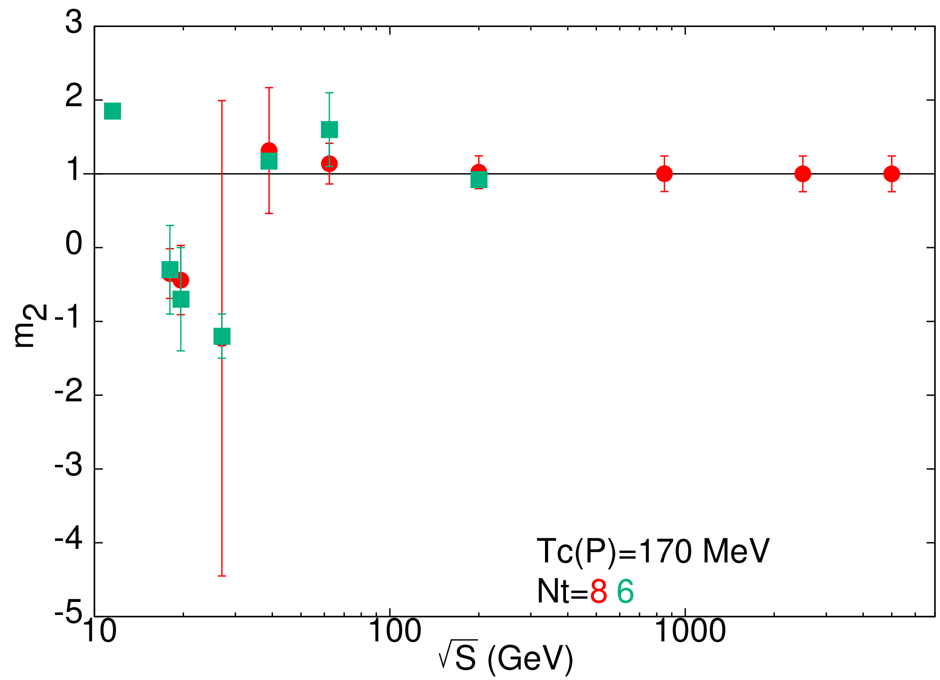
- Leads to a ratio χ_Q:χ_I:χ_B = 1:0:4

- Assuming protons, neutrons, pions to dominate, both χ_Q and χ_B can be shown to be fully reflected in proton number fluctuations.

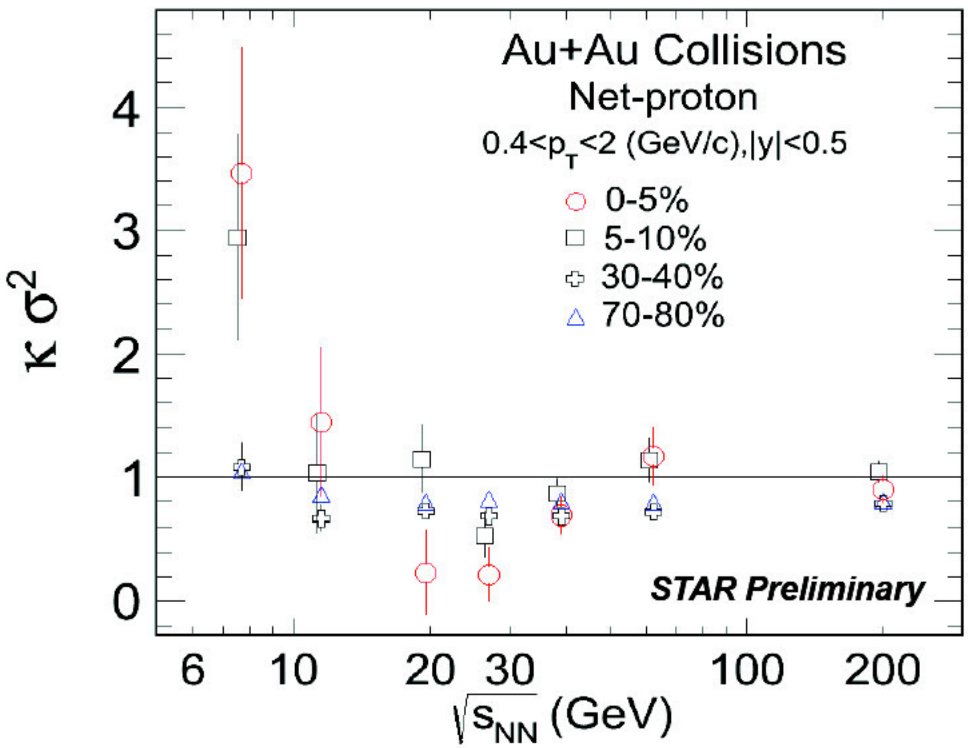




L. Adamcyzk *et al.*
STAR Collaboration PRL (2014)

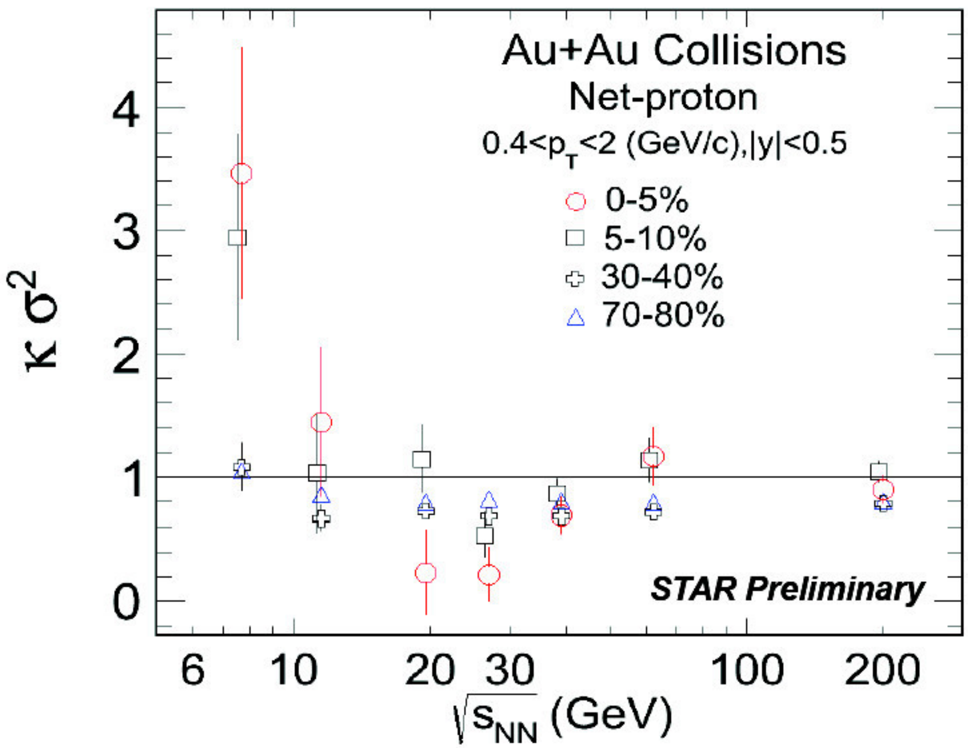
Gavai-Gupta, '10
Datta-Gavai-Gupta, Lattice 2013

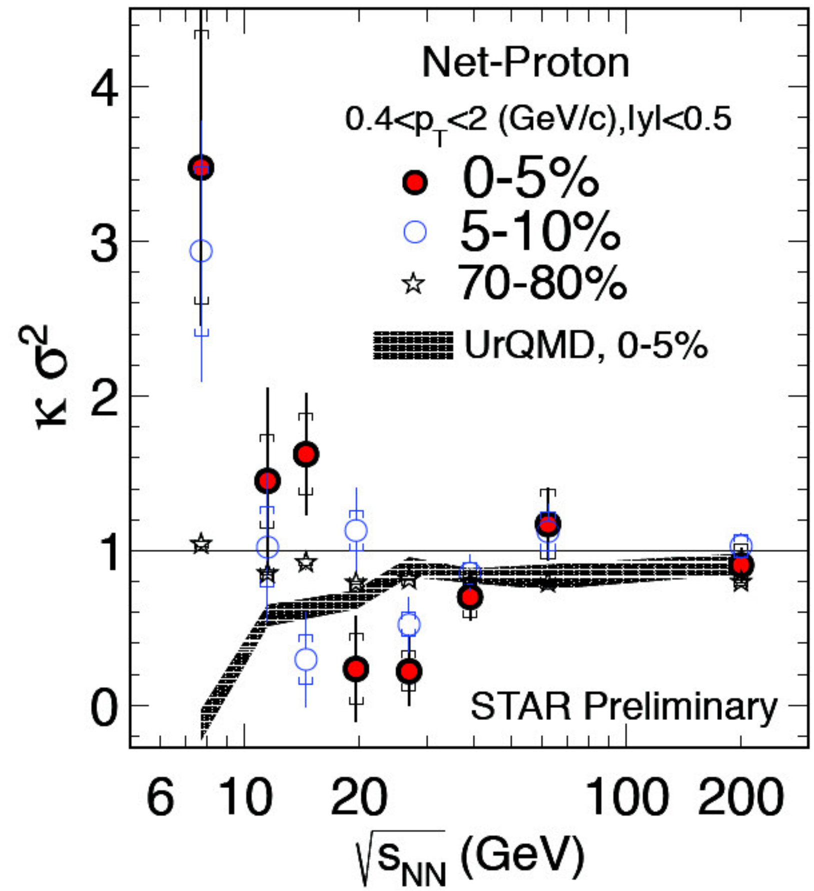
$\mathcal{S}\sigma \equiv m_1$ and $\kappa\sigma^2 \equiv m_2$.



Increasing Δp_T deepens the structure !

X. Luo, CPOD 2014, Bielefeld, STAR Collab.





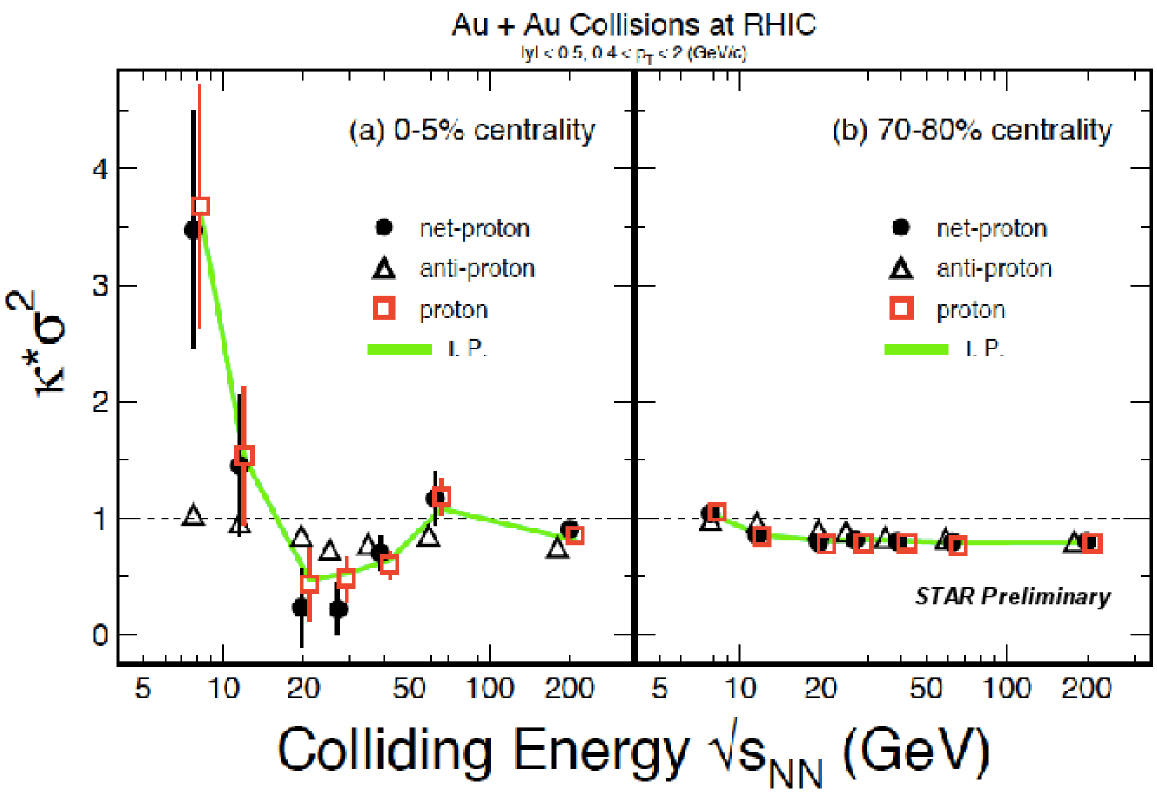
Increasing Δp_T deepens the structure !

X. Luo, CPOD 2014, Bielefeld, STAR Collab.

Interesting Oscillations !!

X. Luo, Quark Matter 2015, Kobe, Japan

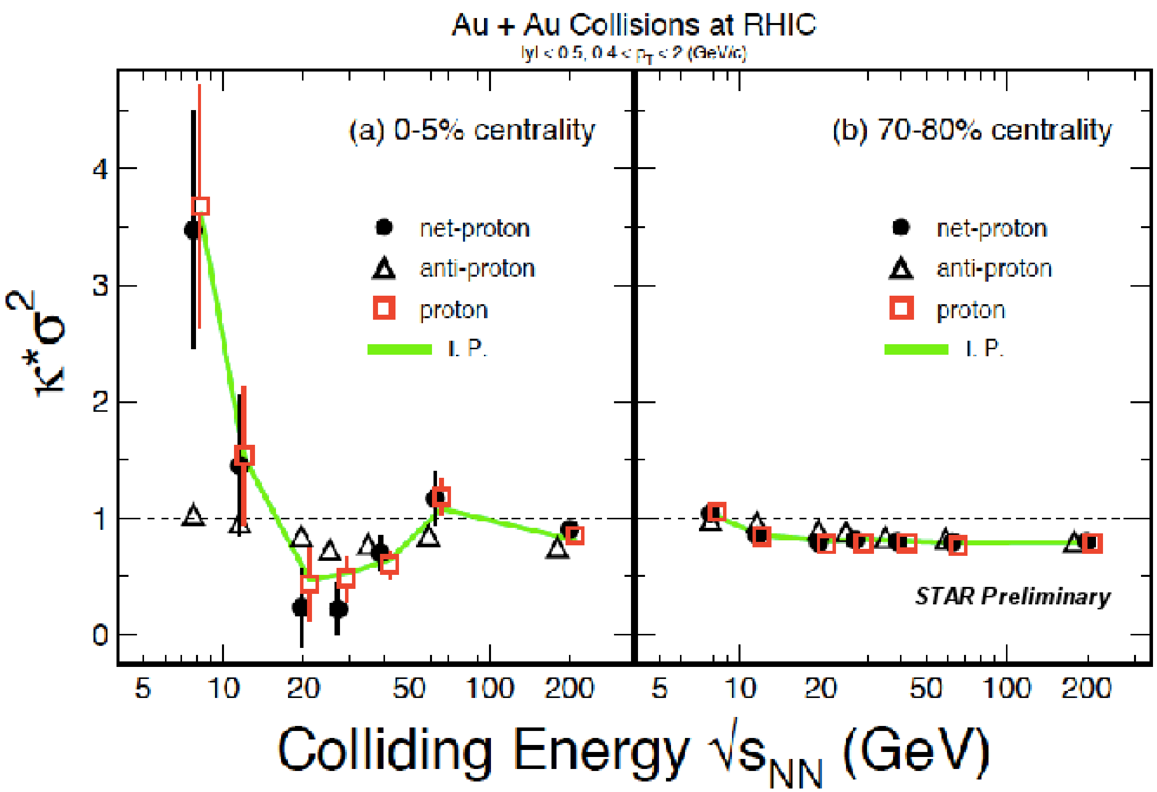
Net-proton Higher Moments



Net-proton results: Non-monotonic behavior in central collision data.

B. Mohanty, xQCD2015

Net-proton Higher Moments



Net-proton results: Non-monotonic behavior in **central collision data.**

B. Mohanty, xQCD2015

"These observables show a centrality and energy dependence, which are neither reproduced by non-CP transport model calculations, nor by a hadron resonance gas model. " — STAR Collaboration PRL (2014).

BES Phase II Proposal

STAR Note 0598

BES Phase II is planned for two 22 cryo-week runs in 2018 and 2019

STAR Upgrades: iTPC, EndCap ToF and Event Plane Detector

$\sqrt{S_{NN}}$ (GeV)	**5.0**	**7.7**	**9.1**	**11.5**	**13.0**	**14.5**	**19.6**
μ_B (MeV)	550	420	370	315	290	250	205
BES I (MEvts)	---	4.3	---	11.7	---	24	36
Rate(MEvts/day)		0.25		1.7		2.4	4.5
BES I L (1×10^{25}/cm^{2}sec)		0.13		1.5		2.1	4.0
BES II (MEvts)		100	160	230	250	300	400
eCooling (Factor)	2	3	4	6	8	11	15
Beam Time (weeks)		**14**	**9.5**	**5.0**	**3.0**	**2.5**	**3.0**

8

Summary

- Phase diagram in $T - \mu$ has begun to emerge: Different methods, $\rightsquigarrow$ similar qualitative picture. Critical Point at $\mu_B/T \sim 1 - 2$.

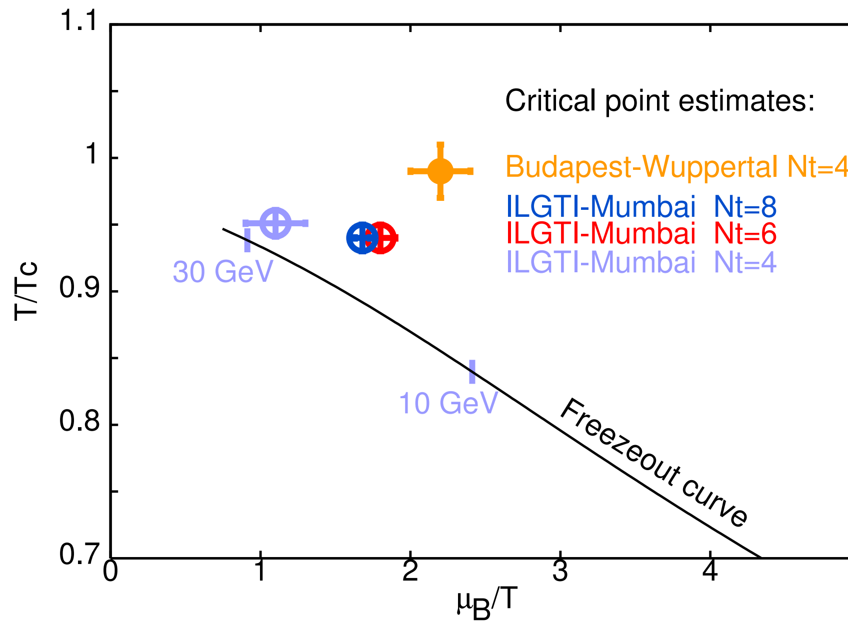
- Our results for $N_t = 8$ first to begin the inching towards continuum limit.

Summary

- Phase diagram in $T - \mu$ has begun to emerge: Different methods, $\rightsquigarrow$ similar qualitative picture. Critical Point at $\mu_B/T \sim 1 - 2$.

- Our results for $N_t = 8$ first to begin the inching towards continuum limit.

- We showed that Critical Point leads to structures in m_i on the Freeze-Out Curve. Possible Signature ?



♡ STAR, BNL results appear to agree with our Lattice QCD predictions. ☺